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ESTIMATION OF THE NUMBER OF FORWARD TIME STEPS FOR THE SEQUENTIAL BECK APPROACH USED FOR SOLVING INVERSE HEAT-CONDUCTION PROBLEMS

UGOTAVLJANJE ŠTEVILA VNAPREJŠNJIH ČASOVNIH KORAKOV ZA SEKVENČNI BECKOV PRIBLIŽEK PRI REŠEVANJU PROBLEMOV INVERZNE TOPLOTNE PREVODNOSTI

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Direct heat-conduction problems are those whose boundary conditions, initial state and material properties are known and the entire temperature field in a model can be computed. In contrast, an inverse problem is defined as the determination of the unknown causes based on the observation of their effects. The inverse heat-conduction method is often used for problems where the boundary conditions cannot be measured directly but are computed from the recorded temperature history inside the model. A very effective method for solving this difficult problem is the sequential Beck approach. To stabilize this inverse problem, a proper regularization parameter must be used. For this method, the regularization parameter is the number of the forward time steps that stabilize the inverse computation. This paper describes two methods for computing the number of the recommended forward time steps for nonlinear heat-conduction models with temperature-dependent material properties. The first method is based on tracking the sensitivity (at the interior point of a measurement) to the Dirac heat-flux pulse on the surface. The second method determines the number of the forward time steps from the residual function computed from the heat fluxes obtained from the inverse computation. The stability and noise (in the results) of several variants of these methods are compared. The results showed that the first method is much less computationally intensive and gives a slightly higher value of the number of forward time steps than the second method.

Keywords: inverse heat-conduction problem, Beck approach, number of forward time steps

Neposredni problemi prevajanja toplote so tisti pri katerih so poznani robni pogoji, začetno stanje in lastnosti materiala ter možnost izračuna temperaturnega polja znotraj modela. Nasprotno pa je inverzni problem definiran kot določanje nepoznanih vzrokov na osnovi opazovanja njihovih vplivov. Metoda inverznega prevajanja toplote se pogosto uporabi pri problemih, kjer se robni pogoji ne morejo neposredno izmeriti, temveč se jih izračuna iz zabeleženega poteka temperature znotraj modela. Zelo učinkovita metoda za reševanje tovrstnega problema je sekvenčni Beckov približek. Za stabilizacijo takšnega inverznega problema se mora uporabiti ustrezen regulirni parameter. Pri tej metodi je regulirni parameter število priporočenih časovnih korakov, ki stabilizirajo inverzni izračun. Članek opisuje dve metodi za izračun števila priporočenih časovnih korakov za nelinearni model prenosa toplote, s temperaturno odvisnimi lastnostmi materiala. Prva metoda temelji na iskanju občutljivosti, na notranji točki merjenja, do Dirac utripa toplotnega toka na površini. Druga metoda določa število vnaprejšnjih časovnih korakov iz preostale funkcije izračunane iz toplotnih tokov, ki so dobljeni z inverznim izračunom. V rezultatih je primerjana stabilnost šuma pri več variantah teh metod. Rezultati so pokazali, da je prva metoda mnogo manj računsko intenzivna in daje rahlo večjo vrednost števila predhodnih časovnih korakov kot druga metoda.

Ključne besede: problem inverzne toplotne prevodnosti, Beckov približek, število vnaprejšnjih časovnih korakov

1 INTRODUCTION

Heat-conduction problems are often solved in engineering applications during simulations. The problem is well known as a direct task. The effect (the temperature field in time) is computed from the causes (the known initial and boundary conditions). Complex direct problems can be solved using many numerical methods such as FDM,[1] FVM,[2] FEM.[3] The situation is opposite for an inverse heat-conduction problem and it is a much more complicated problem. The causes (e.g., the boundary conditions) are determined from the observation of the effects (the temperature record in several points). There are some computational methods dealing with this inverse problem, including the Beck approach,[4] Tikhonov regularization[5] and neural networks.[6] We focus on the sequential Beck approach in this paper.

The basic idea of the sequential approach is to solve the entire task step by step in time. The measured temperature at an interior point at times t_n, t_{n+1},..,$t_{n+N\mathrm{f}}$ is used to compute the heat flux on the boundary at time t_n, where N_f is the number of forward time steps (the regularization parameter). A computation of N_f temperature fields using a direct task is performed for two different values of constant heat fluxes in each time step. The temperature responses from these two direct tasks are compared to the measured temperature. The new value of the heat flux at time t_n is computed. The choice of an appropriate value for N_f is essential for practical computations. A small value leads to instability and a large value smoothes sudden changes in the boundary conditions. Thus, the appropriate value of this parameter is essential.

2 IMPACT OF THE NUMBER OF FORWARD TIME STEPS ON THE COMPUTED RESULTS

The main function of parameter N_f is to guarantee the stability of the computation of this difficult problem. The stability increases with an increasing value of N_f. In **Figure 1**, three results for N_f = 13, 20, 30 are compared to the correct heat-flux record, which was used to generate the input temperature record for the inverse task. The noise was also added to this temperature record (a standard deviation of 0.05 °C). A large oscillation of the computed heat flux for a low value of N_f is obvious. This is mainly due to the added noise in the input data. The noise reduction in the input data is more effective for larger values of N_f (**Figure 1**). This effect indicates that the use of a large N_f is recommended. Unfortunately, increasing N_f has two effects. First, the computation cost is proportional to N_f. A higher value of N_f results in a longer computational time. Second, a large N_f value smoothes the computed results. Abrupt changes as well as the maximum values of the ideal heat flux (**Figure 1**) are significantly reduced when N_f increases. For N_f =30, the computed maximum heat flux is less than 50 % of the ideal heat flux for this test case.

3 METHODS

The appropriate value of forward time steps is different for each computational model. Two types of methods are described in this article to determine its amount. The first, newly proposed, method is based on the temperature response. The idea is to compare two temperature responses at an interior location (usually a

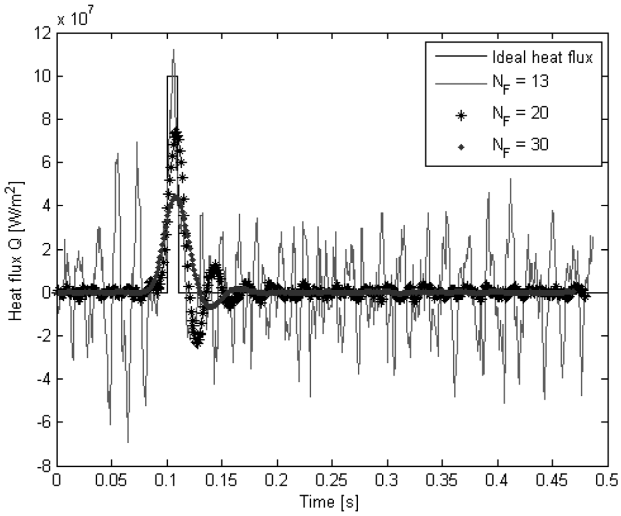

Figure 1: Influence of N_f on the inverse heat-conduction problem
Slika 1: Vpliv N_f na problem inverzne toplotne prevodnosti

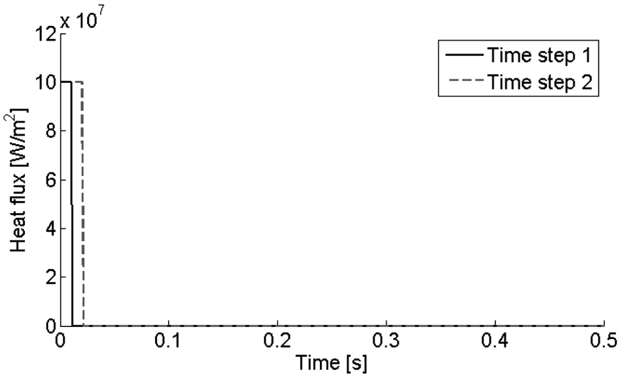

Figure 2: Heat-flux pulses in the subsequent time steps
Slika 2: Sunki toplotnega toka v poznejših časovnih korakih

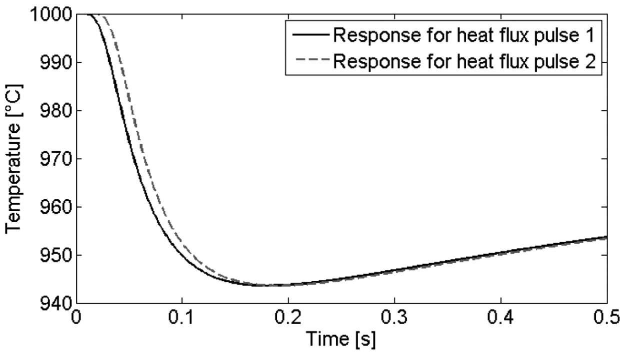

Figure 3: Temperature response
Slika 3: Temperaturni odziv

thermocouple position) to two Dirac pulses of heat flux that are the same but shifted in time by one time step (**Figures 2** and **3**). The first temperature response is computed for the Dirac pulse applied from time step zero to time step one and the second temperature response is computed for the Dirac pulse applied from time step one to time step two. The computed difference between these two temperature responses is shown in **Figure 4**. The computed curve provides an idea of how the information about the changes in the boundary condition is delayed from time step zero to time step two and spread over the time. This curve shows the distribution of the information about the temperature response. This is the information about what happened at the beginning of the

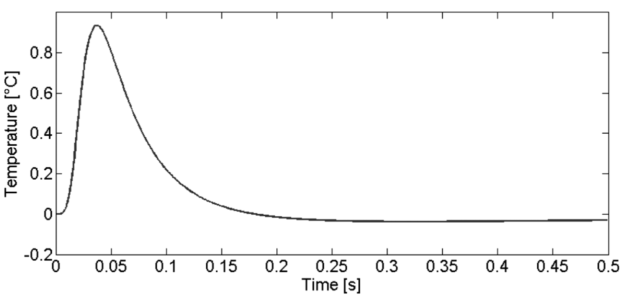

Figure 4: Difference between temperature responses
Slika 4: Razlika med temperaturnimi odzivi

simulation (from time step zero to time step two) at the boundary of the computational model.

For practical computations, it should be noted that both temperature responses are the same except for the time shift, which is one time step. In addition, the temperature response difference $\Delta T_n = T_n - T_{n-1}$ corresponds to the numerical derivation except for the multiplication by constant *c* (Equation (1)):

$$T_n - T_{n-1} = c \cdot \mathrm{d}T = c \cdot \frac{T_n - T_{n-1}}{\Delta t} \quad (1)$$

where $c = \Delta t$.

The shape of the temperature response to the Dirac pulse depends on many parameters. The most important are the material properties (density, thermal conductivity and thermal capacity), the distance of the thermocouple from the boundary, the thermocouple type, the material, and the thermal resistance between the thermocouple and the material.

The number of forward time steps (forward time *t*, respectively) is taken from the derivation of the temperature response $D(t)$ so that $D(t)$ meets a certain criterion.

For example, t_{Dmax} is the time when $D(t_{max})$ is maximal. $t_{Dmax,p\%,1}$ and $t_{Dmax,p\%,2}$ are the times when the derivation of the temperature response reaches *p* % of its maximum. An example for $p = 60$ % is shown in **Figure 5**.

The second estimation method for determining the number of forward time steps can be done with a repeated computation of the inverse heat-conduction problem by changing N_f. The sum of the residuals $R = \Sigma(Q'_i - Q_i)^2$ is evaluated from each inverse task where Q_i is the computed heat flux and Q'_i is the correct heat flux from the test task. An example of how *R* is dependent on N_f is shown in **Figure 6** and the $N_{f,min}$ value (N_f when *R* is minimal) can be found here. The value of N_f (slightly larger than $N_{f,min}$) is taken as an estimate for the number of forward time steps. The $N_{f,min}$ value is not used due to the risk that a small shift of the estimated N_f value to the left (to a smaller value) can rapidly increase the *R* value (**Figure 6**). An analogical application of the

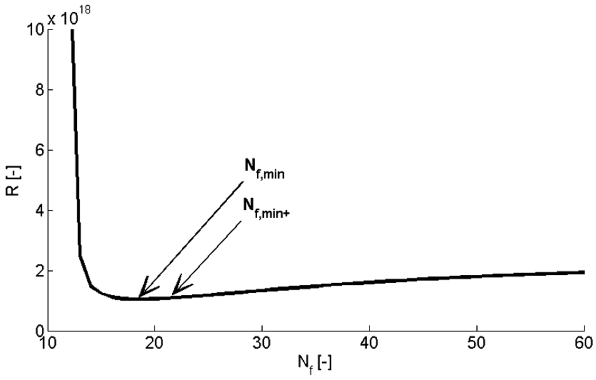

Figure 6: Residual chart for N_f values
Slika 6: Grafikon ostankov za vrednosti N_f

search for the optimum regularization parameter in a Tikhonov digital filter is described by Woodbury.[7]

4 DISCUSSION

The first method described is much less computationally intensive than the second one because the first method needs only one direct computation instead of many inverse (and therefore much more complicated) computations. Each method provides a different value of forward time steps N_f. It is not easy to say which value is better. Generally, this depends on what is more essential for each application. The larger value of N_f smoothes the results but the average values for certain time intervals are correct. A small value of N_f can result in heat fluxes that better fit true values, but the results include more oscillation than would be expected in reality. The choice of the appropriate testing function in the second method also significantly influences the computed value of N_f. Two examples of the testing functions and the obtained N_f value are shown in **Figure 7**.

The comparison of the inverse computations performed with $N_{f,Dmax} = 37$, $N_{f,Dmax,60\%,2} = 65$ (from the first method) and $N_{f,1} = 24$, $N_{f,2} = 18$ (from the second method) is shown in **Figure 8**. The curve for $N_{f,Dmax,60\%,1} = 23$ is not plotted because it is almost the same as that for $N_{f,1} = 24$. These inverse computations

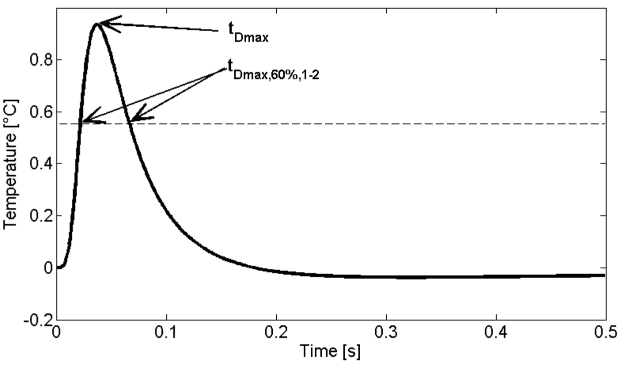

Figure 5: Example of t_{Dmax} and $t_{Dmax,60\%,1-2}$
Slika 5: Primer za t_{Dmax} in $t_{Dmax,60\%,1-2}$

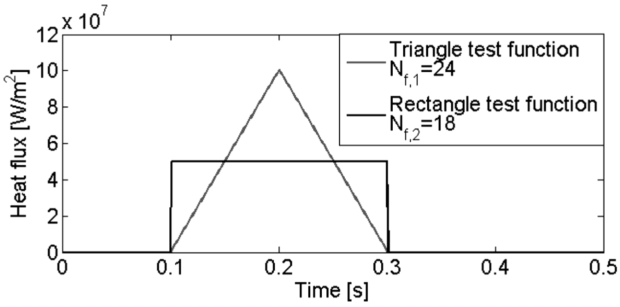

Figure 7: Two examples of testing functions and obtained $N_{f,min}$, using the second method
Slika 7: Dva primera preizkusnih funkcij in dobljen $N_{f,min}$ pri uporabi druge metode

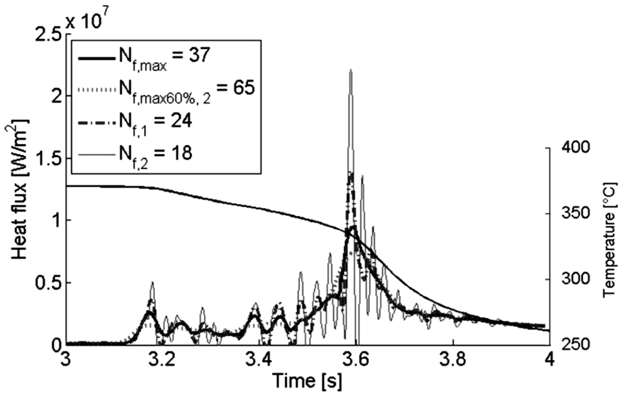

Figure 8: Results of the heat flux for four different values of N_f and the measured temperature

Slika 8: Rezultati toplotnega toka za štiri različne vrednosti N_f in izmerjena temperatura

were made for the 1D inverse heat-conduction problem with thermally dependent material properties. The temperature record from the real measurements was used. Therefore, the correct heat-flux function is unknown.

The heat-conduction problem is described with differential Equation (2):

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \tag{2}$$

where T is the temperature, t is the time and x is the coordinate. The boundary conditions for (Equation (3)) cooled and insulated surfaces are:[4]

$$-k\left.\frac{\partial T}{\partial x}\right|_{x=0} = q(t); \quad -k\left.\frac{\partial T}{\partial x}\right|_{x=l} = 0 \tag{3}$$

The test sample was made from a thick stainless-steel plate (L = 10 mm). One side (x = 0) of the sample was cooled down by water and the other side (x = L) was insulated. A thermocouple was placed under the cooled surface (x = 2 mm).

The curves for N_f = 37 and N_f = 24 (**Figure 8**) appear to be acceptable. The curve for N_f = 65 is too smooth. The curve for N_f = 18 begins to be unstable and the computed heat flux is less than zero for some points, which is physically impossible in this experiment.

5 CONCLUSION

Two methods for determining the number of forward time steps N_f for the sequential Beck approach were described. The first method (based on the derivation of the temperature response to the Dirac heat-flux pulse) is computationally much less intensive. The choice of $N_f = N_{f,Dmax}$ is acceptable for most applications. For some similar tasks, it may be better to use $N_{f,Dmax,p\%}$ with the same suitable value of p.

The second method, which is computationally very intensive, can be useful when the shape of the heat-flux curve is known and the appropriate testing function can be used. The obtained values of N_f were smaller than those computed using the first method and the computed heat fluxes showed more oscillation.

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