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A NEW METHOD FOR ESTIMATING THE HURST EXPONENT *H* FOR 3D OBJECTS

NOVA METODA ZA OCENJEVANJE HURSTOVEGA EKSPONENTA *H* ZA 3D-OBJEKTE

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Mathematics and computer science are very useful in many other sciences. We use a mathematical method, fractal geometry, in engineering, specifically in laser techniques. Characterization of the surface and the interfacial morphology of robot-laser-hardened material is crucial to understand its properties. The surface microstructure of robot-laser-hardened material is rough. We aimed to estimate its surface roughness using the Hurst parameter *H*, which is directly related to the fractal dimension. We researched how the parameters of the robot-laser cell impact on the surface roughness of the hardened specimen. The Hurst exponent is understood as the correlation between the random steps X_1 and X_2, which are followed by time for the time difference Δt. In our research we understood the Hurst exponent *H* to be the correlation between the random steps X_1 and X_2, which are followed by the space for the space difference Δd. We also have a space component. We made test patterns of a standard label on the point robot-laser-hardened materials of DIN standard GGG 60, GGG 60 L, GGG 70, GGG 70 L and 1.7225. We wanted to know how the temperature of point robot-laser hardening impacts on the surface roughness. We developed a new method to estimate the Hurst exponent *H* of a 3D-object. This method we use to calculate the fractal dimension of a 3D-object with the equation $D = 3 - H$.

Keywords: fractal structure, Hurst parameter *H* for 3D-objects, robot, laser, hardening

Matematika in računalništvo sta zelo uporabni veji drugih vrst znanosti. Uporabili bomo matematično metodo – fraktalno geometrijo – v inženirstvu, natančneje, v laserski tehniki. Karakterizacija površine in morfologija robotsko lasersko kaljenih materialov ima ključni pomen za razumevanje lastnosti materialov. Površina robotsko lasersko kaljenega materiala je hrapava. To hrapavost pa bi radi ocenili s Hurstovim eksponentom *H*, ki ga dobimo direktno iz fraktalne geometrije, kot meritev hrapavosti površine. Raziskali smo, kako parameter robotske celice vpliva na hrapavost kaljenih vzorcev. Hurstov eksponent *H* razumemo kot korelacijo med korakoma X_1 in X_2, ki ga dobimo med časovno razliko Δt. V naši raziskavi razumemo Hurstov eksponent *H* kot relacijo med korakoma X_1 in X_2, ki ga dobimo med prostorsko komponento. Osredinili smo se na točkovno robotsko lasersko kaljenje vzorcev standardne oznake po DIN-standardu GGG 60, GGG 60 L, GGG 70, GGG 70 L in 1.7225. Želeli smo izvedeti, kako temperatura točkovnega robotskega laserskega kaljenja vpliva na hrapavost površine. Razvili smo novo metodo za ocenjevanje Hurstovega eksponenta *H* za 3D-objekte. To metodo smo uporabili za računanje fraktalne dimenzije 3D-objektov po formuli $D = 3 - H$.

Ključne besede: fraktalna struktura, Hurstov parameter *H* za 3D-objekte, robot, laser, kaljenje

1 INTRODUCTION

The Hurst parameter[1] is understood as the correlation between the random steps X_1 and X_2, using space for the space difference Δd. It occurs in many areas of applied mathematics, including fractals and chaos theory, and is used in many fields, ranging from biophysics to network computers. The parameter was originally developed in hydrology. However, modern techniques for estimating the Hurst parameter *H* are emerging from fractal mathematics. For example, the fractal dimension was used to measure the roughness of sea coasts. The relationship between the fractal dimension *D* and the Hurst parameter *H* is given by the equation $D = 2 - H$ for 2D-objects and by $D = 3 - H$ for 3D-objects. There is also a form called statistical self-similarity, where if we have one data set of a seemingly endless string of data sets, we can assume that each data set has the same statistical properties as any other. Statistical self-similarity occurs in a surprisingly large number of areas in engineering. Fractal geometry[2,3] has offered a new perspective on maths and science and allows observation of the world around us from a new and completely different point of view. Nature is full of shapes and images that from a distance are seen to be similar. A well-known example of such a self-similar pattern is Sierpinski's pyramid. Let us assume that we have a statistically self-similar finite data series. Any part of this series would have the same statistical characteristics as the entire series or any other part of this series. Such series are often used in engineering. We used one in the analysis of robot-laser-hardened materials.[4] A robot-laser surface-hardening heat treatment is complementary to conventional flame or inductive hardening. The energy source for the laser hardening is a laser beam, which heats up very quickly. Laser hardening is a process of controlled energy intake, high performance constancy, and an accurate positioning process. A hard martensitic microstructure provides improved surface properties such as wear resistance and high strength. Point robot-laser hardening[5–8] is a classic case of robot-laser hardening. This means that the speed of the laser beam is no longer a parameter that can be changed. In point robot-laser hardening cells we are interested in finding the optimal parameters that give the maximum hardness of the hardened material. In this work we have used a scanning electronic microscope (SEM) to search and analyse the fractal structure of the robot-laser-hardened material. First, we introduce the Hurst parameter to find how the temperature of the robot laser cell impacts on the optimal Hurst parameter *H* of the hardened material. We then developed a new method to estimate the Hurst parameter *H* of a 3D-object.

2 MATERIALS PREPARATION AND METHOD

2.1 Materials preparation

We made test patterns of a standard label on point robot-laser-hardened materials of DIN-standard GGG 60, GGG 60 L, GGG 70, GGG 70 L and 1.7225. We hardened the materials at different temperatures. So we changed the temperature parameter of the robot laser cells, $T \in [800, 2000]$ °C, with a step of 100 °C. In all the experiments we recorded the microstructure. We wanted to know how the temperature of point robot-laser hardening impacts on the surface roughness. **Figure 1** shows the transverse and longitudinal section of a hardened material. Each sample was etched and polished (IMT, Institute of Metals and Technology Ljubljana, Slovenia) and then examined under a microscope (IJS, Jožef Stefan Institute). The images were obtained using a JEOL JMS-7600F field-emission scanning electron microscopy. **Figures 2** to **6** present microstructure of robot laser hardened specimens. **Figure 7** presents boundary between hardened and non-hardened DIN 1.7225 standard material.

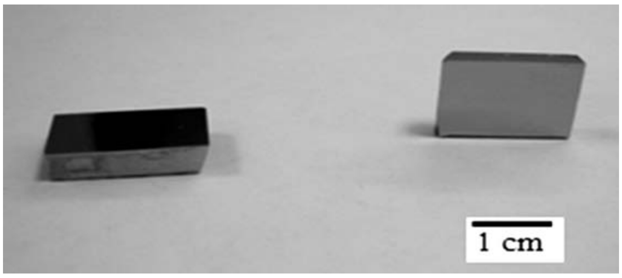

Figure 1: Transverse and longitudinal section of hardened material
Slika 1: Prečni in vzdolžni prerez kaljenega materiala

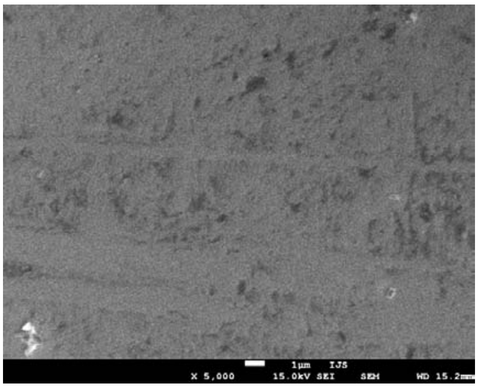

Figure 2: Microstructure of hardened DIN 1.7225 standard material (SEM)
Slika 2: Mikrostruktura kaljenega materiala po DIN-standardu 1.7225 (SEM)

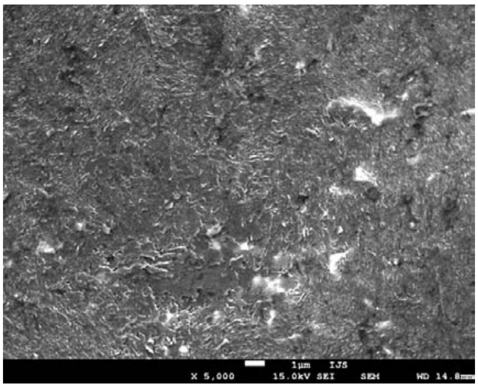

Figure 3: Microstructure of hardened DIN GGG 70 standard material (SEM)
Slika 3: Mikrostruktura kaljenega materiala po DIN-standardu GGG 70 (SEM)

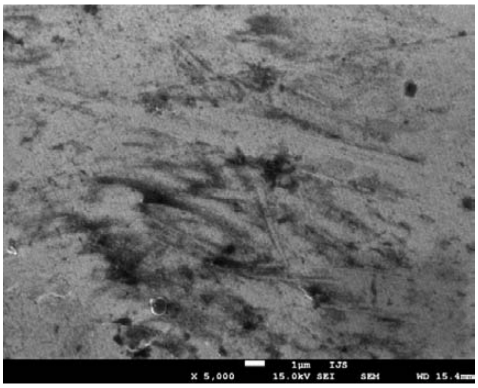

Figure 4: Microstructure of hardened DIN GGG 70 L standard material (SEM)
Slika 4: Mikrostruktura kaljenega materiala po DIN-standardu GGG 70 L (SEM)

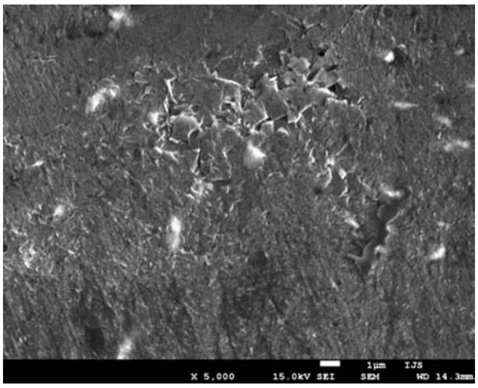

Figure 5: Microstructure of hardened DIN GGG 60L standard material (SEM)
Slika 5: Mikrostruktura kaljenega materiala po DIN-standardu GGG 60 L (SEM)

The random process is evaluated statistically using the Hurst parameter H or by determining the distribution function. The Hurst parameter H as a self-similarity criterion cannot be accurately calculated; it can only be estimated. There are several different methods[9–13] for producing estimates of the parameter H, which deviate from one another to some extent. In doing so, we have no criteria to determine which method gives the best result.

Different methods for estimating the Hurst exponent H have been evaluated.[14] The assessment methods in the space component domain are based on a comparison of the original process and the average process with the method of aggregation:

- The variance-time plot analysis is based on the property of the slowly decaying variance of self-similar processes undergoing aggregation.
- The R/S method. The adjusted rescaled range method or adjusted scale is also a graphical method based on the properties of the Hurst phenomenon.
- In statistics, residual variance is another name for an unexplained variation, the sum of squares of differences between the y-value of each ordered pair on the regression line and each corresponding predicted y-value; it is generally used to calculate the standard error of an estimate. In other words, residual variance helps us confirm how well the regression line that we constructed fits the actual dataset. The smaller the variance, the more accurate the predictions are.

Methods for evaluation in the frequency or wavelet ("wavelet") space are:

- The periodogram method based on the noise $1/f$ and the Fourier transform.
- The Whittle estimator is based on minimizing the likelihood function used in the periodogram method. There is no graphical method.
- The Hurst exponent is estimated by using the wavelet transform of the series. A least-squares fit on the average of the squares of the wavelet coefficients at different scales is an estimate of the Hurst exponent. The method produces both a graphical output and a confidence interval.

Estimation of the Hurst parameter H[15–19] using different methods gives results that show significant differences.

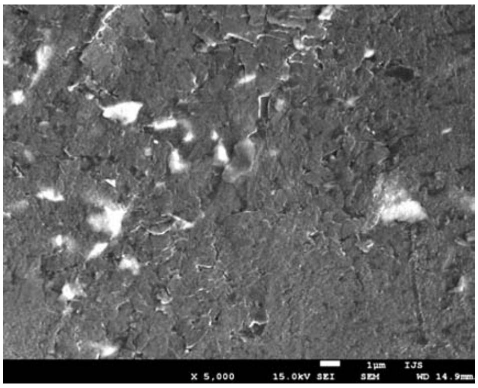

Figure 6: Microstructure of hardened DIN GGG 60 standard material (SEM)

Slika 6: Mikrostruktura kaljenega materiala po DIN-standardu GGG 60 (SEM)

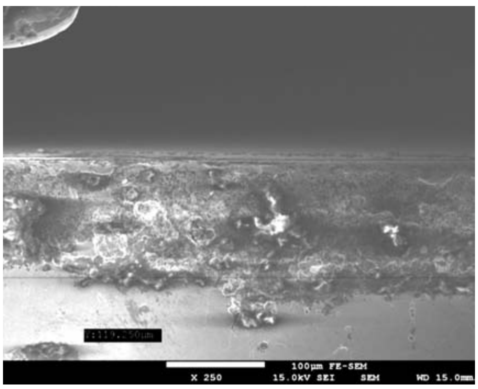

Figure 7: The boundary between hardened and non-hardened DIN 1.7225 standard material (SEM)

Slika 7: Meja med kaljenim in nekaljenim delom materiala po DIN-standardu 1.7225 (SEM)

2.2 Method

Here we present our first new method for estimating the Hurst exponent H for 3D-objects. First of all, we find all the coordinates (x, y, z) of an SEM picture. Here, we use the program ImageJ. Then we use only z coordinates to estimate the Hurst exponent H. We present all the z coordinates in a 2D-space component graph (**Figure 8**), which is continuous. Also, all the points (x_i, y_0, z_i) present the first space component in a 2D-graph for all the

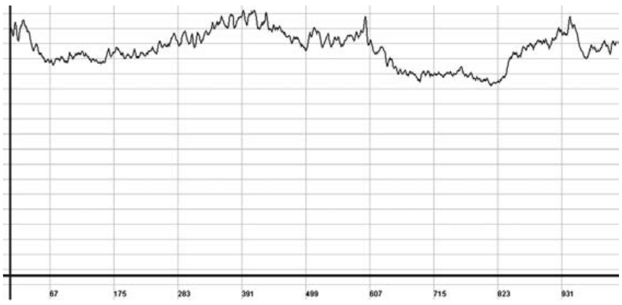

Figure 8: Space component

Slika 8: Prostorska komponenta

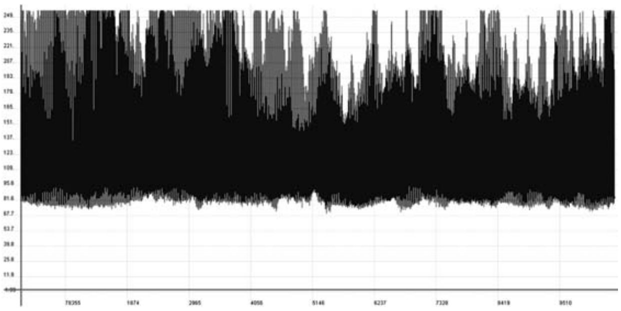

Figure 9: Long space component

Slika 9: Dolga prostorska komponenta

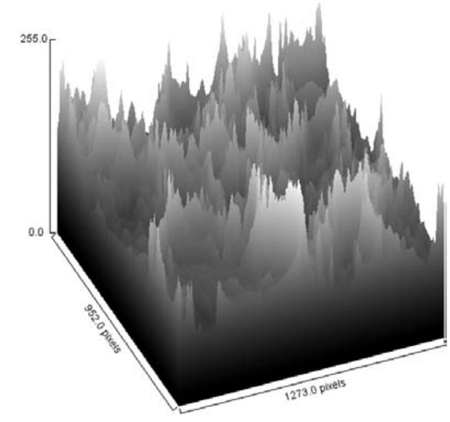

Figure 10: 3D-object of robot-laser-hardened microstructure
Slika 10: 3D-objekt lasersko kaljene mikrostrukture

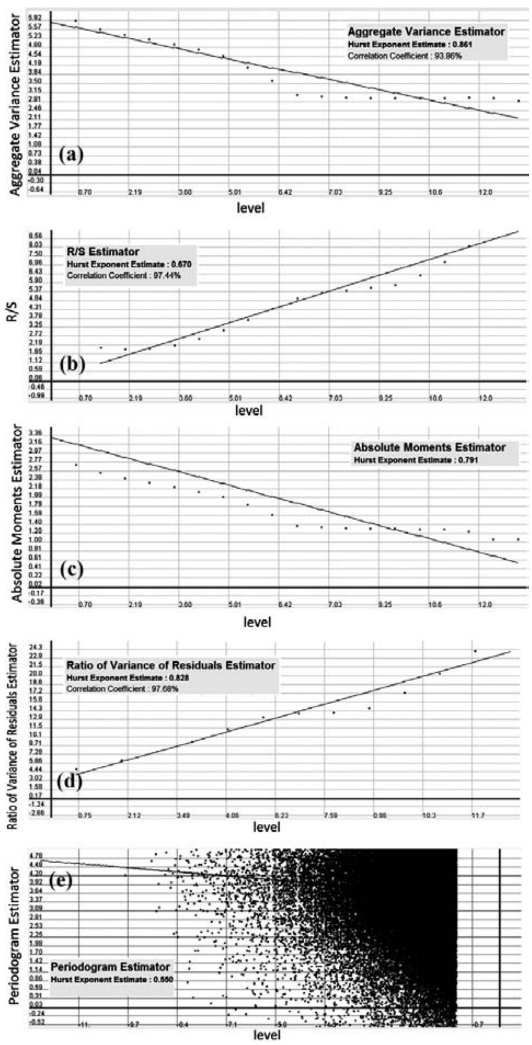

Figure 11: a) Variance method for hardened material 1.7225, b) R/S method for hardened material 1.7225, c) Absolute moments estimator for hardened material 1.7225, d) Variance of residuals method for hardened material 1.7225, e) Periodogram method for hardened material 1.7225
Slika 11: a) Variance metoda za kaljeni material 1.7225, b) R/S-metoda za kaljeni material 1.7225, c) Ocena absolutnega momenta za kaljeni material 1.7225, d) Variančna metoda ostankov za kaljeni material 1.7225, e) Metoda periodograma za kaljeni material 1.7225

points (x_i, z_i). All the points (x_i, y_1, z_i) represent the second space component in a 2D-graph for all the points (x_i, z_i). We obtained the space component for all y_i, $\forall i$ (**Figure 9**). Then we combined all of these space components into one space component. For this long space component we can estimate the Hurst exponent *H*. **Figure 10** shows the 3D-object of the point robot-laser-hardened microstructure.

3 RESULTS AND DISCUSSION

The pictures in the .jpeg format were converted into 256-grey-level numerical matrices (level 1 for black and 256 for white) with the program ImageJ. Then we entered information into the program Selfis 01B, with which we obtained the following graphs. The graphs were made only for specimen 1.7225 hardened at 800 °C.

Figures 11a to **11d** show the different methods for estimating the Hurst exponent *H*.

Figures 12 to **16** present the estimated value of the Hurst parameter *H* with five methods for five different robot-laser-hardened materials.

The collected data were entered into the program and the graphs were obtained, from which we can deduce the optimum results.

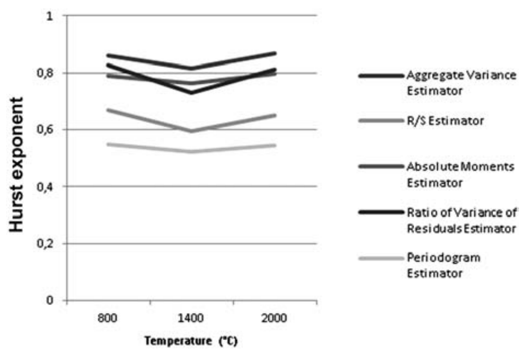

Figure 12: Relationship between estimated value of the Hurst parameter *H* and temperature of robot-laser-hardened specimens for DIN standard 1.7225
Slika 12: Odvisnost med ocenjeno vrednostjo Hurstovega parametra *H* in temperaturo robotsko lasersko kaljenih vzorcev 1.7225

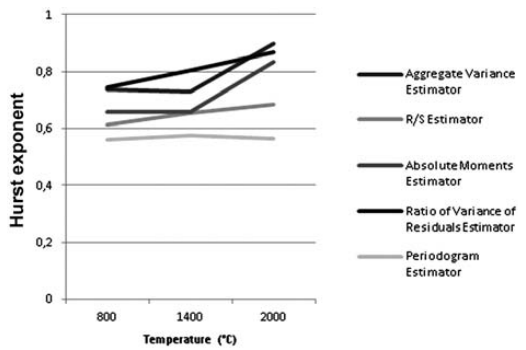

Figure 13: Relationship between estimated value of the Hurst parameter *H* and temperature of robot-laser-hardened specimens for DIN standard GGG 70
Slika 13: Odvisnost med ocenjeno vrednostjo Hurstovega parametra *H* in temperaturo robotsko lasersko kaljenih vzorcev GGG 70

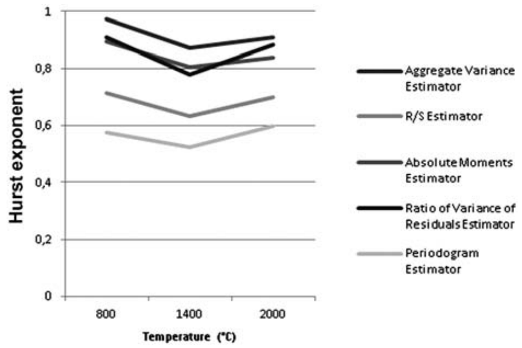

Figure 14: Relationship between estimated value of the Hurst parameter *H* and temperature of robot-laser-hardened specimens for DIN standard GGG 70 L

Slika 14: Odvisnost med ocenjeno vrednostjo Hurstovega parametra *H* in temperaturo robotsko lasersko kaljenih vzorcev GGG 70 L

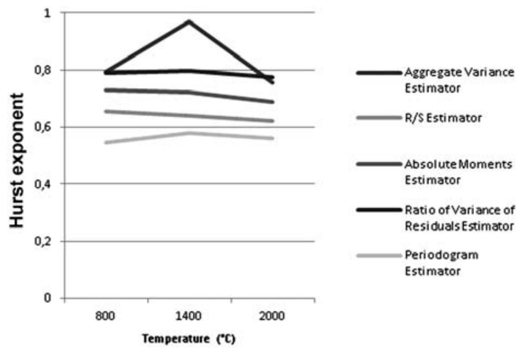

Figure 15: Relationship between estimated value of the Hurst parameter *H* and temperature of robot-laser-hardened specimens for DIN standard GGG 60 L

Slika 15: Odvisnost med ocenjeno vrednostjo Hurstovega parametra *H* in temperaturo robotsko lasersko kaljenih vzorcev GGG 60 L

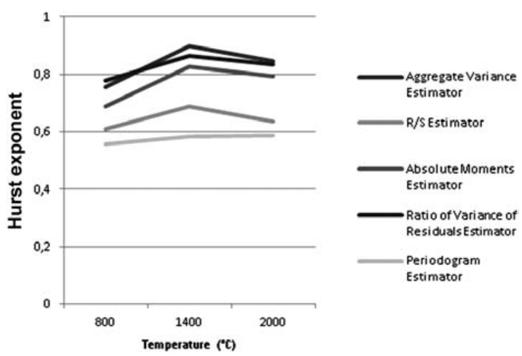

Figure 16: Relationship between estimated value of the Hurst parameter *H* and temperature of robot-laser-hardened specimens for DIN-standard GGG 60

Slika 16: Odvisnost med ocenjeno vrednostjo Hurstovega parametra *H* in temperaturo robotsko lasersko kaljenih vzorcev GGG 60

Material 1.7225: The smallest Hurst exponent *H* is obtained at a temperature of 1400 °C with all methods, which means that the roughness is higher.

Material GGG 70: The highest Hurst exponent *H* is obtained at a temperature of 2000 °C with all methods, which means that the roughness is lower.

Material GGG 70 L: The smallest Hurst exponent *H* is obtained at a temperature of 1400 °C with all methods, which means that the roughness is higher. It is similar to the material 1.7225.

Material GGG 60 L: The smallest Hurst exponent *H* is obtained at a temperature of 2000 °C with all methods, without for the periodogram method.

Material GGG 60: The smallest Hurst exponent *H* is obtained at a temperature of 800 °C with all methods, which means that the roughness is higher. The highest Hurst exponent *H* is obtained at a temperature of 1400 °C with all methods without for the periodogram method.

Smaller values are obtained with the periodogram method for all the hardened specimens at all temperatures. Higher values are obtained with the aggregate variance estimator method for hardened specimens at all the temperatures.

4 CONCLUSIONS

We conducted experiments on five different materials. We changed only the temperature parameter, $T \in [800, 2000]$ °C, in 100 °C steps. In total there were 65 samples. We were interested in which parameter of the robot laser cell achieved the roughest surface of the hardened material. We found the Hurst parameter *H* exactly. The main findings can be summarized with the following points:

- A fractal structure exists in robot-laser hardening.
- The Hurst parameter *H* was calculated using different methods. The results were compared using the program Image J and it was found that the best results were obtained by the periodogram method, which gave smaller values.
- With the help of the equation $D = 2 - H$, the fractal dimensions can be calculated for all the samples. For calculating the fractal dimensions in the 2D-plane obtained by a microscope as a two-dimensional image from which the fractal dimension can be determined.
- In the 3D-space we use equation $D = 3 - H$. In addition, we found a new process for calculating the fractal dimension of a 3D object.
- We found the optimal Hurst parameter *H* for different laser parameters of robot cells on different materials.
- The smaller the Hurst parameter *H*, the greater the surface roughness.
- We developed a new method to estimate the Hurst parameter *H* of a 3D-object.

The author of the Selfis program has written a great deal about the Hurst parameter *H*. The problem with these methods is that we estimated only the Hurst parameter *H* and that these methods are based on different bases (some of aggregation, others on wavelet transformation, etc.), which in turn lead to different estimates. Also, each method has certain advantages and limitations with regard to the captured sample (some methods are better for large samples, some for smaller ones). Many books state that there is no precise method with which to accurately calculate the Hurst parameter and that it can only be estimated. The most common method of calculating the Hurst parameter H is the R/S-method.

Our findings are important from a practical point of view. The precise parameters of the robot-laser cell tempering influence the hardness of the hardened material. Materials with such properties have better wear resistance and a longer lifespan. The Hurst parameter H can give us information about the correlation between the roughness and the hardness of material, which is important in many industries.

Acknowledgements

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5 FUTURE WORK PLAN

In the future we plan to explore the Hurst parameter H as a function of several parameters in robot cells for laser hardening. The laser parameters include power, energy density, focal distance, energy density in the focus, focal position, temperature, and speed of hardening. In robot-laser hardening, many different problems are encountered. We are interested in estimating the Hurst parameter H in:

- two-beam laser robot hardening (where the laser beam is divided into two parts);
- areas of overlap (where the laser beam covers an already hardened area);
- robot-laser hardening at different angles (the angles change depending on the x and y axes).

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