NUMERICAL STUDY OF HEAT-TRANSFER ENHANCEMENT OF HOMOGENEOUS WATER-Au NANOFLUID UNDER NATURAL CONVECTION

NUMERIČNA ANALIZA POVEČANJA PRENOSA TOPLOTE HOMOGENE NANOTEKOČINE VODA-Au POD POGOJI NARAVNE KONVEKCIJE

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A numerical analysis is performed to examine the heat transfer of colloidal dispersions of Au nanoparticles in water (Au nanofluids). The analysis used a two-dimensional enclosure under natural convection heat-transfer conditions and has been carried out for the Rayleigh number in the range of $10^3 \leq Ra \leq 10^5$, and for the Au nanoparticles' volume-fraction range of $0 \leq \varphi \leq 0.10$.
We report highly accurate numerical results indicating clearly that the mean Nusselt number is an increasing function of both Rayleigh number and volume fraction of Au nanoparticles. The results also indicate that a heat-transfer enhancement is possible using nanofluids in comparison to conventional fluids. However, low Rayleigh numbers show more enhancement compared to high Rayleigh numbers.

Keywords: natural convection, water-Au nanofluid, heat transfer, numerical modelling

V prispevku smo numerično analizirali prenos toplote v koloidnih disperzijah nanodelcev zlata v vodi (Au-nanotekočine). Pri tem smo obravnavali dvodimenzionalno kotanjo pod pogoji naravne konvekcije za vrednosti Rayleighevega števila $10^3 \leq Ra \leq 10^5$ in volumenske koncentracije Au-nanodelcev $0 \leq \varphi \leq 0,10$.
Rezultati analize kažejo, da je srednje Nusseltovo število naraščajoča funkcija obeh, tako Rayleighevega števila kot volumenskega deleža Au-nanodelcev. Prikazani rezultati nakazujejo, da lahko prenos toplote izboljšamo z uporabo Au-nanotekočin namesto navadnih tekočin. Pri tem pa je pozitivni učinek na prenos toplote izrazitejši pri nižjih vrednostih Rayleighevega števila.

Ključne besede: naravna konvekcija, nanotekočina voda-Au, prenos toplote, numerično modeliranje

1 INTRODUCTION

Today more than ever, ultra-high-performance heat transfer plays an important role in the development of energy-efficient heat-transfer fluids required in many industries and commercial applications. However, conventional heat-transfer fluids (e.g. water, oil or ethylene glycol) are inherently poor heat transfer fluids. Nanofluid, a term coined by Choi[1] in 1995, is a new class of heat-transfer fluids developed by suspending nanoparticles, such as small amounts of metal, non-metal or nanotubes in the fluids. The goal of nanofluids is to achieve the highest possible thermal properties at the smallest possible volume concentrations with a uniform dispersion and a stable suspension of nanoparticles in host fluids.

Buoyancy-induced flow and heat transfer is an important phenomenon used in various engineering systems. Some applications are solar thermal receivers, vapour absorption refrigerator units[2] and electronic cooling, selective laser melting processes[3], etc. Several researchers have been focused on numerical modelling of such flows. Oztop and Abu-Nada[4] studied the two-dimensional natural convection of various nanofluids in partially heated rectangular cavities and reported that the type of nanofluid is the key factor for a heat-transfer enhancement. They obtained the best results with Cu nanoparticles. Hwang et al.[5] studied natural convection of a water-based Al_2O_3 nanofluid in a rectangular cavity heated from below. They investigated the convective instability of the flow and heat transfer and reported that the natural convection of the nanofluid becomes more stable when the volume fraction of nanoparticles increases. Ho et al.[6] studied the effects on the nanofluid heat transfer caused by viscosity and thermal conductivity in a buoyant enclosure. They demonstrated that the usage of different models for viscosity and thermal conductivity has a major impact on the heat transfer and flow characteristics.

The effect of the inclination angle on the heat-transfer enhancement under natural convection has been studied by Oztop et al.[7] (for water-based Al_2O_3 and TiO_2 nanofluids) and by Abu-Nada and Oztop[8] (for water-based Cu nanofluids). They reported that the effect of the

inclination angle on the percentage of a heat-transfer enhancement becomes insignificant at a low Rayleigh number, but it decreases the enhancement of heat transfer with a nanofluid. Last but not least, the inclination angle is reported to be a good control parameter for both pure and nanofluid-filled enclosures.

Although quite some work has been done in this area, it is still safe to conclude that there is a lack of numerical studies of the heat characteristics of the nanofluids containing Au nanoparticles. The present work is therefore directed to study the natural convection heat-transfer characteristics of the water-based Au nanofluids for the Rayleigh number in the range of $10^3 \leq Ra \leq 10^5$ and for the volume fraction of $0 \leq \varphi \leq 0.10$.

2 NUMERICAL MODELLING

The standard finite volume method is used to solve the coupled conservation equations of mass, momentum and energy. This method has been used successfully in a number of recent studies to simulate generalized New-tonian fluid flows[9,10]. In this framework a second-order central differencing scheme is used for the diffusive terms and a third-order QUICK scheme for the convective terms. The coupling of the pressure and velocity is achieved using the well-known SIMPLE algorithm. The convergence criteria were set to 10^{-8} for all residuals.

2.1 Governing equations

For the present study, a steady-state flow of an incompressible water-based Au nanofluid is considered. It is assumed that both the fluid phase and the nano-particles are in thermal equilibrium. Except for the density, the properties of the nanoparticles and the fluid are taken to be constant. **Table 1** presents the thermo-physical properties of water and gold at the reference temperature. It is further assumed that the Boussinesq approximation is valid for the buoyancy force.

The governing equations (mass, momentum and energy conservation) for a steady, two-dimensional laminar and incompressible flow are:

$$\frac{\partial v_i}{\partial x_i} = 0 \qquad (1)$$

$$\rho_{nf} v_j \frac{\partial v_i}{\partial x_j} - \frac{\partial}{\partial x_j}\left(\eta_{nf} \frac{\partial v_i}{\partial x_j}\right) =$$

$$= -\frac{\partial p}{\partial x_i} + (\rho\beta)_{nf} g(T - T_C) + \frac{\partial}{\partial x_j}\left(\eta_{nf} \frac{\partial v_j}{\partial x_i}\right) \qquad (2)$$

$$(\rho c_p)_{nf} v_j \frac{\partial T}{\partial x_j} = \frac{\partial}{\partial x_j}\left(c_{nf} \frac{\partial T}{\partial x_j}\right) \qquad (3)$$

where the cold wall temperature T_C is taken to be the reference temperature for evaluating the buoyancy term $(\rho\beta)_{nf}g(T - T_C)$ in the momentum conservation equation.

The relationships between the properties of the nanofluid (*nf*) and those of the pure fluid (*f*) and the pure solid (*s*) are given with the following empirical models[7]:

• Density:

$$\rho_{nf} = (1-\varphi)\rho_f + \varphi\rho_s$$

• Dynamic viscosity:

$$\eta_{nf} = \frac{\eta_f}{(1-\varphi)^{2.5}}$$

• Thermal expansion:

$$(\rho\beta)_{nf} = (1-\varphi)(\rho\beta)_f + \varphi(\rho\beta)_s$$

• Heat capacitance:

$$(\rho c_p)_{nf} = (1-\varphi)(\rho c_p)_f + \varphi(\rho c_p)_s$$

• Thermal conductivity:

$$k_{nf} = k_f \frac{k_s + 2k_f - 2\varphi(k_f - k_s)}{k_s + 2k_f + \varphi(k_f - k_s)}$$

Table 1: Thermo-physical properties of the Au nanofluid
Tabela 1: Termo-fizikalne lastnosti Au-nanotekočine

	ρ (kg/m³)	c_p (J/kg K)	k (W/m K)	β (1/K)
Pure water	997.1	4179	0.613	2.1×10^{-4}
Au	19320	128.8	314.4	1.416×10^{-7}

2.2 Geometry and boundary conditions

The simulation domain and the expected temperature distribution are shown schematically in **Figure 1**. The two vertical walls of the square enclosure are kept at different constant temperatures ($T_H - T_C$), whereas the other boundaries are considered to be adiabatic in nature. Both velocity components (i.e., v_x and v_y) are identically zero on each boundary because of the no-slip condition and impenetrability of the rigid boundaries. The temperatures for cold and hot vertical walls are specified (i.e. $T(x = 0) = T_H$ and $T(x = L) = T_C$). The adiabatic temperature boundary conditions for the horizontal insulated boundaries are given by $\partial T/\partial y = 0$ at $y = 0$ and $y = L$.

In the present study, the heat-transfer rates (along the hot vertical wall) in a square enclosure (of the dimension
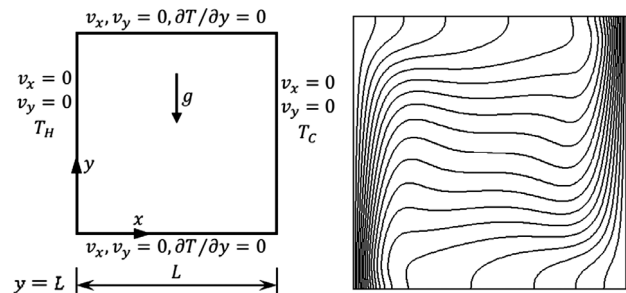

Figure 1: Schematic diagrams of the simulation domain (left) and the expected temperature field (right)
Slika 1: Shematični prikaz območja simulacije (levo) in pričakovano temperaturno polje (desno)

L), with differentially heated side walls, filled with Au nanofluid are expressed in terms of the local and mean Nusselt number as follows:

$$Nu(y) = \frac{k_{nf}}{k_f} \frac{\partial T(y)}{\partial x}\bigg|_{x=0} \frac{L}{T_{x=0} - T_C} \qquad (4)$$

$$\overline{Nu} = \int_0^L \frac{Nu(y)\mathrm{d}y}{L} \qquad (5)$$

and compared with the heat-transfer rate obtained in the case of pure water ($\varphi = 0$) with the same nominal Rayleigh number. Here the Rayleigh number Ra represents the ratio of the strengths of the thermal transports due to buoyancy to the thermal diffusion and is defined in the following manner:

$$Ra = \frac{\rho_{nf} (\rho c_p)_{nf} \, g\beta_{nf} \, (T_H - T_C)L^3}{\eta_{nf} \, k_{nf}} \qquad (6)$$

2.3 Grid-dependency study

The grid independence of the results has been established on the basis of a detailed analysis of three different uniform meshes: $M1(50 \times 50)$, $M2(100 \times 100)$ and $M3(200 \times 200)$. For the general primitive variable ϕ the grid-converged (i.e. extrapolated to the zero element size) value, according to Richardson extrapolation, is given as[9,10]:

$$\phi_{ext} = \phi_{M3} - \frac{(\phi_{M2} - \phi_{M3})}{(r^p - 1)}$$

where ϕ_{M3} is obtained on the basis of the finest grid, ϕ_{M2} is the solution based on the next level of coarse grid, $r = 2$ is the ratio between the coarse and fine grid spacing and $p = 2.88$ is the actual order of accuracy.

Table 2: Effect of mesh refinement upon the mean Nusselt number ($\varphi = 0$, $Ra = 10^5$)
Tabela 2: Vpliv zgoščevanja mreže na srednjo vrednost Nusseltovega števila ($\varphi = 0$, $Ra = 10^5$)

Mesh $M1$	Mesh $M2$	Mesh $M3$	$\overline{Nu}_{ext}$	e
4.704	4.721	4.723	4.724	0.008 %

The numerical error $e = \left|(\phi_{M3} - \phi_{ext})/\phi_{ext}\right|$ for the mean Nusselt number $\overline{Nu}$ is presented in **Table 2**. It can be seen that the differences with grid refinement are exceedingly small and the agreement between mesh $M3$ and extrapolated value is extremely good; the discretisation error for $\overline{Nu}$ is well below 0.01 %. Based on this the simulations in the remainder of the paper were conducted on mesh $M3$ which provided a reasonable compromise between high accuracy and computational efficiency.

2.4 Benchmark comparison

In addition to the aforementioned grid-dependency study, the simulation results have also been compared with the well-known benchmark data of de Vahl Davis[11] relating to natural convection of air ($Pr = 0.71$) in a square cavity for the values of the Rayleigh number $10^3 \leq Ra \leq 10^5$. The comparisons between the present simulation results with the corresponding benchmark values are extremely good and entirely consistent with our grid-dependency studies. The comparison is summarised in **Table 3**.

Table 3: Comparison of the present results with the benchmark results
Tabela 3: Primerjava dobljenih rezultatov z referenčnimi

	$Ra = 10^3$		$Ra = 10^4$		$Ra = 10^5$	
	Nu_{max}	$\overline{Nu}$	Nu_{max}	$\overline{Nu}$	Nu_{max}	$\overline{Nu}$
Present study	1.506	1.118	3.531	2.245	7.722	4.521
de Vahl Davis	1.505	1.118	3.528	2.243	7.717	4.519

3 RESULTS AND DISCUSSION

Figure 2 presents a variation of the local Nusselt number (Equation 4) along the hot wall for different values of Ra. In the conduction dominated heat-transfer mechanism, **Figure 2a**, the variation of the local Nu is similar for all solid volume fractions. Up to $y/L \approx 0.10$ it is characterized with a constant value and with a further increase in the y/L local Nusselt number decreases. In addition, it can be observed that the values of Nu along the whole hot wall are greater in the case of a higher volume fraction of the Au nanoparticles.

As the Ra increases, **Figure 2b**, the maximum of the local Nusselt number (Nu_{max}) is shifted away from the
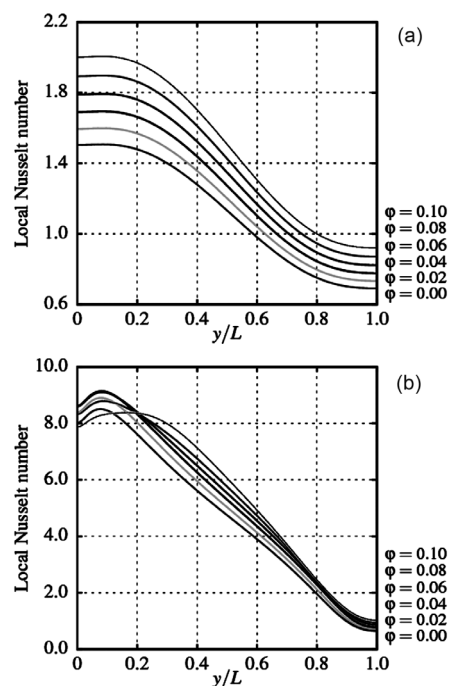
Figure 2: Variation of the local Nusselt number along the hot wall for: a) $Ra = 10^3$ and b) $Ra = 10^5$
Slika 2: Spreminjanje lokalnega Nusseltovega števila vzdolž tople stene za: a) $Ra = 10^3$ in b) $Ra = 10^5$
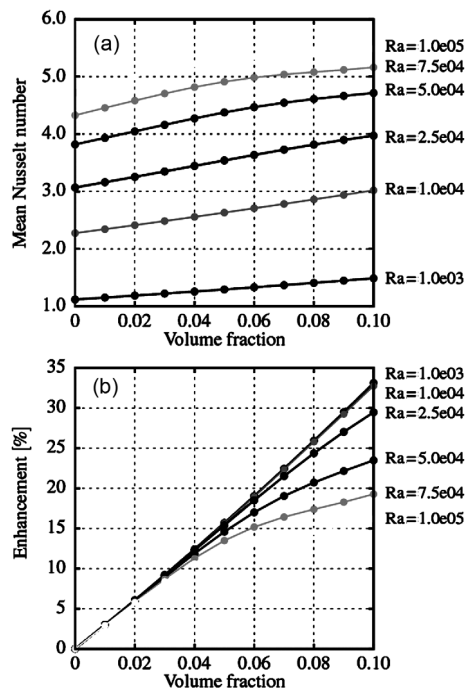
Figure 3: a) Variation of the mean Nusselt number along the hot wall and b) the heat-transfer enhancement due to an addition of Au nanoparticles

Slika 3: a) Spreminjanje srednjega Nusseltovega števila vzdolž tople stene in b) povečanje prenosa toplote zaradi dodanih Au-nanodelcev

bottom adiabatic wall and is an increasing function of the solid volume fraction up to $\varphi = 0.04$. With the higher values of the solid volume fraction, Nu_{max} starts to decrease and its location is shifted further away from the bottom adiabatic wall.

Regardless of the Ra value, the decrease in Nu is more pronounced in the $0.2 < y/L < 0.8$ region and it attains the minimum value at the upper wall ($y/L = 1.0$).

The variation of the mean Nusselt number (Equation 5) along the hot wall with a solid volume fraction is shown in **Figure 3a** indicating that $\overline{Nu}$ increases with an increasing φ. For $Ra \leq 10^4$, where the heat transfer is conduction dominated, the distribution of $\overline{Nu}$ is completely linear. The distribution of the mean Nusselt number becomes increasingly non-linear with the strengthening of the convective transport in the cases of higher values of Ra for all volume fractions of Au nanoparticles.

The previous discussions indicate that, generally, heat transfer is enhanced with an addition of nanoparticles. To estimate the enhancement of the heat transfer in the case of Au nanofluid and in the case of pure fluid ($\varphi = 0$), the enhancement is defined[6]:

$$E = \frac{\overline{Nu}(\varphi) - \overline{Nu}(\varphi = 0)}{\overline{Nu}(\varphi = 0)} \times 100\% \qquad (6)$$

The enhancement of heat transfer is plotted with respect to the Au nanoparticles volume fraction at different Rayleigh numbers as shown in **Figure 3b**. Consider-

ing the whole range of Rayleigh numbers, the figure illustrates that heat transfer increases in the case of an increasing solid volume fraction φ. It is interesting to observe that the heat-transfer enhancement is an increasing linear function of the volume fraction in the cases of the lower values of the Rayleigh number ($Ra \leq 10^4$), while the higher values of the Rayleigh number ($Ra > 10^4$) are characterized with a non-linear increase in the heat-transfer enhancement.

Finally, the enhancement of the heat transfer for $\varphi \leq 0.03$ is similar for all the values of Ra and as the volume fraction further increases, the heat transfer is greater with the low Rayleigh numbers than with the high Rayleigh numbers. This is related to the difference between the conduction dominated mechanism for the heat transfer at a low Ra and the convection mechanism at a high Ra.

4 CONCLUSIONS

In the present study, the heat-transfer characteristics of the steady laminar natural-convection water-based Au nanofluids in a square enclosure with differentially heated side walls have been numerically studied. The effects of the Rayleigh number ($10^3 \leq Ra \leq 10^5$) and the solid-volume fraction ($0 \leq \varphi \leq 0,10$) have been systematically investigated.

The influence of computational grid refinement on the present numerical predictions was studied throughout the examination of the grid convergence for the natural convection at $Ra = 10^5$. By utilizing extremely fine meshes, the resulting discretisation error for $\overline{Nu}$ is well below 0.01 %.

The numerical method was validated for the case of the convection of air ($Pr = 0.71$) in a square cavity, and its results are available in the open literature. A remarkable agreement of our results with the benchmark results of de Vahl Davis[11] yields sufficient confidence in the present numerical procedure and its results.

The highly accurate numerical results confirmed some important points, such as:

- Both the increasing value of the Rayleigh number and the solid-volume fraction of the nanoparticles augment the heat-transfer rate (the mean Nusselt number).
- The mean Nusselt number $\overline{Nu}$ is an increasing function of both, the Rayleigh number Ra and the volume fraction φ of the Au nanoparticles.
- The effect of the highly conductive nanoparticles on the heat-transfer enhancement is more significant at the low values of the Rayleigh number (the conduction-dominated heat transfer).

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