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SIMILARITY CRITERIA AND EFFECT OF LUBRICANT INERTIA AT COLD ROLLING

MERILA PODOBNOSTI IN VPLIV VZTRAJNOSTI MAZIVA PRI HLADNEM VALJANJU

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Modern rolling mills operate at speed requiring the consideration of inertial forces for the explanation of the behaviour of lubricant layer in the enter gap of the metal deformation zone. For this calculation a similarity criterium and the improved Mizun-Grudev equation were applied. Good results were obtained in comparison with the numerical Monte Carlo method. A significant point of the calculation is the definition of the initial state in the point of singularity.

Key words: cold rolling, lubrification, lubricant inertia, gripping angle, Reynolds equation, Monte Carlo method

Moderne valjarne obratujejo pri hitrostih valjanja, ki zahtevajo upoštevanje sile vztrajnosti za razlago vedenja plasti maziva na vhodni reži zone deformacije metala. Za izračun sta bili uporabljeni merilo podobnosti in Mizum-Gridevova enačba. Dobljeni so dobri rezultati v primerjavi z numerično metodo Monte Carlo. Pomembna točka izračuna je definicija začetnega stanja v točki singularnosti.

Ključne besede: hladno valjanje, mazanje, vztrajnost maziva, kot oprijema, Reynoldsova enačba, metoda Monte Carlo

1 INTRODUCTION

The lubricant achieves in the enter gap of the metal deformation[1–5] zone a wedge shape determined with the geometry of the rolls and the rolled sheet, as shown in **Figure 1**. The flow of lubricant in the rolling gap can be described with the simplified Reynol's differential [6–9] equations:

$$\frac{\partial p}{\partial x}=\mu\frac{\partial^2 v_x}{\partial y^2} \tag{1a}$$

$$\frac{\partial p}{\partial y}=0 \tag{1b}$$

$$\frac{\partial v_x}{\partial x}+\frac{\partial^2 v_y}{\partial y}=0 \tag{1c}$$

$$Z=-\int\frac{\partial v_x}{\partial x}\mathrm{d}y+C(x) \tag{1d}$$

According to the differential equation (1b), the pressure in the lubricant layer is constant over the gap height and that it changes along the layer length, only, and the approximate analytical solution of equation (1a) is:

$$v_x=\frac{1}{\mu}\frac{dp}{dx}\frac{y^2}{2}+C_1(y)+C_2 \tag{1e}$$

Assuming boundary conditions from **Figure 1**

$$\begin{matrix} v_x=v_0 & y=0 \\ v_x=v_R & y=\varepsilon(x) \end{matrix} \tag{1f}$$

the integration constants are:

$$C_1=\frac{v_{Rx}-v_0}{\varepsilon(x)}-\frac{1}{\mu}\frac{dp}{dx}\frac{\varepsilon(x)}{2} \tag{1g}$$

$$C_2=v_0 \tag{1h}$$

Including (1g) and (1h) in (1d) we obtain

$$v_x=\frac{1}{2\mu}\frac{dp}{dx}\left[y^2-\varepsilon(x)y\right]+\left[\frac{v_{Rx}-v_0}{\varepsilon(x)}\right]y+v_0 \tag{1i}$$

The lubricant consumption along the strip perimeter is:

$$Q(x)=\int_0^{\varepsilon(x)}v_x\,dy=-\frac{1}{12\mu}\frac{dp}{dx}\varepsilon(x)+\left[\frac{v_0+v_{Rx}}{2}\right]\varepsilon(x) \tag{1j}$$

For $x = 0$ $$Q=\left(\frac{v_0+v_R}{2}\right)\varepsilon_0 \tag{1k}$$

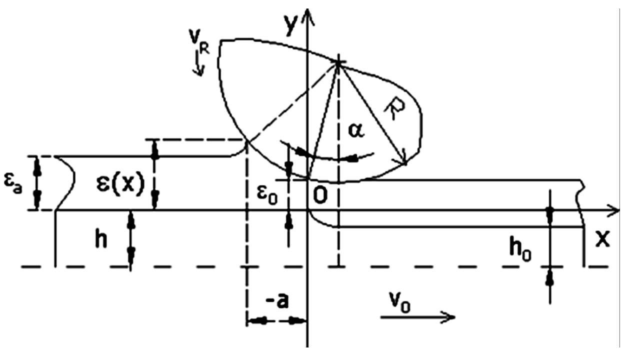

Figure 1: Scheme of cold rolling with lubricant
Slika 1: Shema hladnega valjanja z mazivom

Equalizing (1j) and (1k) we obtain[3–9]:

$$\frac{dp}{dx}=\frac{6\mu\left(v_0+v_R\right)}{\varepsilon^2(x)}-\frac{12\mu Q}{\varepsilon^2(x)} \tag{1l}$$

The equation was solved applying numerical methods[10,11] for dressing rolling including inertial forces. The solution is:

$$\frac{dp}{dx}=\frac{6\mu\left(v_0+v_R\right)}{\varepsilon^2(x)}+\frac{C_1\mu}{\varepsilon^2(x)}+ +\frac{x\rho}{120R\varepsilon^3(x)}\left[16(v_0+v_R)^2\varepsilon^2(x)-C_1^2\right] \tag{2}$$

$$C_1=\frac{k}{2}-\sqrt{\frac{k^2}{4}+2(v_0+v_R)\varepsilon_0\left[8(v_0+v_R)\varepsilon_0+3k\right]} \tag{3}$$

$$\frac{k=120vR}{x} \tag{4}$$

The lubricant thickness in the gap range (–a, 0) is:

$$\varepsilon(x)=\varepsilon_0+R\left\{\cos\alpha-\sqrt{1-\left(\sin\alpha-\frac{x}{R}\right)^2}\right\} \tag{5a}$$

This equation can be developed in the power series:

$$\varepsilon(x)=\varepsilon_0-\varepsilon x+\frac{1}{2R}x^2-\frac{1}{2R^2}x^3+\ldots \tag{5b}$$

Neglecting inertia forces, analytical solutions can be developped in the form:

$$(\varepsilon_0)^d=\varepsilon_0^1-\left[(\varepsilon_0^1-\varepsilon_0^1)/\alpha^*\right]\alpha \tag{6}$$

$$\mathrm{A}=\frac{-\alpha}{2\varepsilon_0\psi}+\left[\frac{1}{R\psi x^{1/2}}\right]\Omega+\frac{3\alpha}{2R\psi^2}-\left[3\varepsilon_0/2R^2\psi^2\xi^{1/2}\right]\Omega \tag{7}$$

With: R/m – rolls diameter, v_0 and v_R/(m/s) – rolling speeds, μ_0/(Pa s) – dynamical viscosity of the lubricant, ρ/(kg/m^3) – fluid (lubricant) density, v/(m^2/s) – kinematical viscosity of the lubricant, α (rad) – rolling grip angle, A/m^{-1} – technological parameter, ε_0/m – lubricant layer thickness in the initial section of metal deformation zone, ε^1_0/m – lubricant layer thickness for $\alpha\to 0$, v_x and v_y/(m/s) – speeds in Descartes coordinates x and y, γ/(m^2/N) – pieso coefficient of lubricant viscosity, ε_0^* – lubricant layer thickness in the singularity point α^*, p_0/(N/m^2) – rolling pressure. The parametric symbols are explained in **Table 1**.

These equations can be used for calculations and computer modelling of the behaviour of the lubricant layer in the zone of plastic deformation[1,10,11,12] of metals with cold rolling.

The characteristics of thin sheets rolling process allow to find analytical solutions for equation (1l), however, it is difficult to find a solution for equation (2). For lubricated rolling of sheets, the analytical solutions are acceptable for high gripping angles, while simulation models are mostly used for continous rolling [12,13].

Table 1: Parametrical symbols for **Figure 1** and equations (1) to (7)
Tabela 1: Parametrični simboli za **sliko 1** in enačbe od (1) do (7)

ε_0^1	$(\pi^2R/128A^2)^{1/3}$
ε_0^*	$(1/2)R(\alpha*)^2$
α^*	$(8/15RA)^{1/3}$
D	Square determinant of equation (6) for the members ($\alpha*$: ε_0^*)
ζ	$-\psi$
ψ	$(2/R)\varepsilon_0-\alpha^2$
Ω	$\ln[-\alpha-(\zeta)^{1/2}/(-\alpha+(\zeta)^{1/2}]$
A	$[(1-\exp(-\gamma p_0))/6\mu_0\gamma(v_0+v_R)]$
Linear interaction	$\alpha^*\cong 1.246(\varepsilon_0^1/R)^{1/2}$; $\varepsilon_0^*\cong 0.7726\,\varepsilon_0^1$; $\varepsilon_0^*=R(\alpha^*)^2/2$
Mizun-Grudev equation	$\varepsilon^M{}_0=1/2A\alpha$
$(\varepsilon_0)^d$	Linearisation for the dressing rolling

2 SIMILARITY CRITERIA

Similarity criteria are frequently used in fluid mecanics calculations. Basic data for calculation related to the behaviour of the lubricant in the metal deformation zone in **Table** 2 for the term $\varepsilon^M{}_0$ and for the transcendent equation (7) were deduced for the following processing conditions: A = 1.965512 · 10^6 m^{-1}, v_0 = 6 m/s, v_R = 10 m/s, $\alpha_i=(m)\pm^{(1/3)}\alpha$. The iso values are based on investigations aimed to obtain a better equation than that of Mizunov-Grudev given in **Table 2** and analytically more acceptable than the transcendent equation (7). The

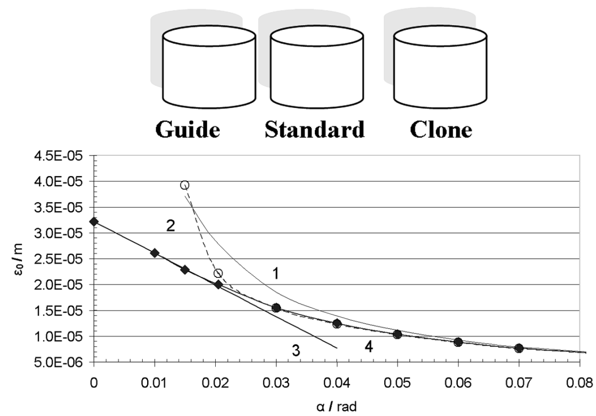

Figure 2: Comparison of different methods for the calculation of the lubricant layer thickness (R = 0.35 m, A = 898519 m^{-1}. 1 – Mizunov-Grudev equation, 2. o – improved Mizunov-Grudev equation (8), 3 – method of linearisation of the equation (6), 4. ♦ – numerical Monte Carlo solution.

Guide	R = 0.35 m	A = 1965512 m^{-1}
Standard	R = 0.35 m	A = 800 000 m^{-1}
Clone	R = 0.35m	A = 898 519 m^{-1}

Slika 2: Primerjava različnih metod za izračun debeline plasti maziva (R = 0.35m, A = 898 519 m^{-1}. 1 – Mizunov-Grudev enačba, 2. o – izboljšana Mizunov-Grudev enačba (8), 3 – metoda of linearizacije enačbe (6), 4. ♦ – numerična Monte Carlo rešitev

Vodilo	R = 0.35 m	A = 1 965 512 m^{-1}
Standard	R = 0.35 m	A = 800 000 m^{-1}
Klon	R = 0.35m	A = 898 519 m^{-1}

Table 2: Etalon (standard) for theoretical investigations of similarity criteria ($m = R/R_i$)
Tabela 2: Etalon (standard) za teoretično raziskavo meril podobnosti ($m = R/R_i$)

R/m	Rolling grip angle/rad							
	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08
0.05		1.725186	1.385324	1.234872	1.155365	1.108717	1.079329	1.059809
0.1		1.510205	1.259770	1.153034	1.098537	1.067490	1.048406	1.035988
0.15		1.410961	1.203689	1.117577	1.074549	1.050460	1.035857	1.026462
0.2		1.350680	1.170418	1.096966	1.060832	1.040848	1.028848	1.021185
0.25		1.309130	1.147908	1.083234	1.051802	1.034578	1.024310	1.017789
0.30	1.833992	1.278294	1.131459	1.073322	1.045345	1.030127	1.021105	1.015400
0.35	1.774128	1.254262	1.118808	1.065776	1.040466	1.026783	1.018708	1.013621
0.40	1.725186	1.234872	1.108718	1.059809	1.036633	1.024169	1.016841	1.012238
0.45	1.684164	1.218813	1.100447	1.054955	1.033532	1.022062	1.015341	1.011130
0.50	1.649108	1.205241	1.093520	1.050917	1.030965	1.020324	1.014107	1.010220
0.55	1.618683	1.193583	1.087620	1.047498	1.028800	1.010886	1.013072	1.009458
0.60	1.591942	1.183434	1.082523	1.044561	1.026947	1.017615	1.012190	1.008810
-	-	-	-	-	-	-	-	-
0.8	1.510205	1.153034	1.067490	1.035988				

Table 3: Comparison of different methods to the Monte-Carlo method for bridging the problematic area in **Figure 3** for a = 0.02 to a = 0.035 rad
Tabela 3: Primerjava različnih metod z metodo Monte-Carlo za premostitve problematične površine na **sliki 3** za a = 0.02 do a = 0.035 rad

Gripping angle / rad	Equation (6)	Method of linearisation	Monte-Carlo	Equation (8)
α = 0.02	$19.909 \cdot 10^{-6}$	$19.949 \cdot 10^{-6}$	$19.916 \cdot 10^{-6}$	-
α = 0.025	-	$17.532 \cdot 10^{-6}$	$17.499 \cdot 10^{-6}$	-
α = 0.03	-	$15.536 \cdot 10^{-6}$	$15.509 \cdot 10^{-6}$	$15.471 \cdot 10^{-6}$
α = 0.035	-	$13.884 \cdot 10^{-6}$	$13.866 \cdot 10^{-6}$	$13.716 \cdot 10^{-6}$
	Second part			
α = 0.04	-	$12.304 \cdot 10^{-6}$	$12.501 \cdot 10^{-6}$	$12.341 \cdot 10^{-6}$
α = 0.045	-	$11.342 \cdot 10^{-6}$	$11.353 \cdot 10^{-6}$	$11.215 \cdot 10^{-6}$
α = 0.05	-	$10.352 \cdot 10^{-6}$	$10.387 \cdot 10^{-6}$	$10.271 \cdot 10^{-6}$
α = 0.055	-	-	$9.560 \cdot 10^{-6}$	$9.467 \cdot 10^{-6}$
	Third part			
		Derivation method	Monte-Carlo	Equation (8)
α = 0.07		$7.884 \cdot 10^{-6}$	$7.686 \cdot 10^{-6}$	$7.639 \cdot 10^{-6}$
α = 0.08		$6.950 \cdot 10^{-6}$	$6.784 \cdot 10^{-6}$	$6.754 \cdot 10^{-6}$
α = 0.09		$6.224 \cdot 10^{-6}$	$6.067 \cdot 10^{-6}$	$6.046 \cdot 10^{-6}$

authors have developed the following aproximation for this equation:

$$\left(\frac{4V-10}{\alpha R}\right)(\varepsilon_0^w)^2 + 2\alpha\varepsilon_0^w - A^{-1} = 0$$

$$V = \ln\left(\frac{2\alpha^2\,\mathrm{Re}}{\varepsilon_0^M}\right) \tag{8}$$

With: e – natural logarithm base.

Tests have shown that the error is for equation (8) in comparison to equation (7) smaller than 1 % for gripping angles $\alpha > 0.03$ rad. Further, equation (8) preserves the same law of iso values than the Mizunov-Grudev equation. It follows, that by applying the criteria of iso values it is possible by cold rolling to pass over from a greater to a smaller gripping angle and to calculate the lubricant layer thickness according to equation (8) using the relation $\alpha_i = (m)^{\pm(1/3)}\alpha$. The values in **Table 2** cannot be calculated using equation (1) developed to a polynome of the third or higher order but only to the square polynome according to equation (5b).

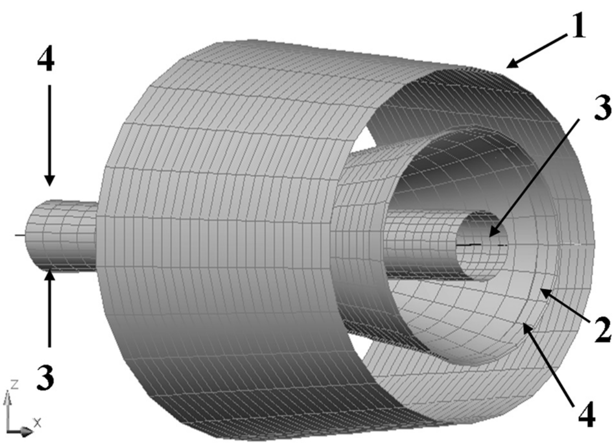

Figure 3: Modelled solutions for **Figure 2** at α = 0.03 rad. Same denotations as in **Figure 2**. The tool REVSURF allows the rotation around the linearisatioin method (curve 3).
Slika 3: Modelirane rešitve za **sliko 2** pri α = 0.03 rad. Iste označbe kot na **sliki 2**. Orodje REVSURF omogoča rotacijo okoli metode linearizacije (krivulja 3).

In **Figure 2** the comparison of results of the calculation of the lubricant layer thickness according to different methods is shown. The figures show that the linearisation method can be applied only for the dressing rolling. For cold rolling the Mizunov-Grudev solution differs from the numerical solution for aproximately α = 0.06 rad, while equation (8) differs for only α = 0.025 rad. For the range α = 0.02 to α = 0.03 rad on **Figure 2** the criterium of similarity can be used and an approximate solution can be obtained with interpolation from known values.

The criterium of similarity is based on equation (6) and the selected transfer function was a hyperbola. The thickness of the lubricant layer is calculated using the linearisation method and applying the standard data in **Table 2** over the guider to the case on **Figure 2**. The hyperbola is used to transmit the lubricant layer thickness defined by the equation (8), as in this case, or using the boundary conditions for a = 0.02 and a = 0.03, as shown in **Table 3**. In this table the comparison is given of the lubricant layer thickness calculated using the similarity criteria and the Monte Carlo method for the gripping angles indicated. The guider is defined for R = 0.35 and $A = 8 \cdot 10^5$ m^{-1}. It is clear from **Table 3** that the methods of linearisation according to equation (6), of similarity criteria and of the improved Mizunov-Grudev equation are complementar and represent an algorithm for the thickness of the lubricant layer suitable for fast practical application.

In **Figure 3** the AutoCAD modelling of **Figure 2** using different modelling methods is shown. In the initial part of abscissa a good approximation is obtained between the numerical and the linearisation method and that in the border part of abscissa with α = 0.03 the agreement between the corrected equation (9) and the numerical Monte-Carlo calculation is acceptable. Straight lines obtained with linear programming and derivation at the point of singularity are shown in **Figure 4**. The linear representation was selected to obtain the optimal transfer of similarity criteria to the cylinder-clone for which the solution of differential equation is obtained using the known solutions for the guider and the standard on the base of only one solution for the singular point ($\alpha*$, ε^*_0).

This approach is valid for gripping angles in the range 0.0124 rad to 0.0404 rad with allowed angle change for $\Delta\alpha \approx 0.00362$ rad. In the second part of **Table 3** the transfer of similarity criteria from the standard over the guider to the cylinder clone using the straight lines 4, 5 and 6 in **Figure 5**, is shown. If the strainght lines transferring the accuracy from the first to the fourth quadrant become unreliable for the transfer of similarity criteria, it is possible to use, as help, the singular point derivation with direction of transfer of similarity criteria to the first quadrant for which the calculation of the lubricant layer thickness is performed. The singular point transfers well the calculation of the lubricant film, although the gripping angle is relatively distant. It is usefull to remind that the method of derivation in **Table 3** offers the possibility of increasing the mathematical accuracy in comparison to the Monte-Carlo method. In this case, the simple analytical definition would approach a form approaching equation (8). Further, data in **Table 3** show that the improved equation (8) can, for smooth surfaces of rolls and of rolled metal, substitute the Monte Carlo method. In this case, the transfer of similarity criteria for the standard and the guider can be calculated using equation (8), while the clone cylinder is defined mathematically with differential equations connecting the longitudinal and transverse roughness and with possible analytical solutions for the point of singularity.

The solution of differential equations related to the equation (1a), as for instance equation (2), can be obtained through the solution for one point, as boundary condition, in this case the solution for the singular point. If the accuracy as in **Table 1** is expected, the calculation of the lubricant layer for smooth surfaces can not be simplified further.

Using a complex method of statistical analysis of data in **Table 2** and for the rolls of diameter of R = 0.3 m the following relation was developed for the lubricant layer thickness:

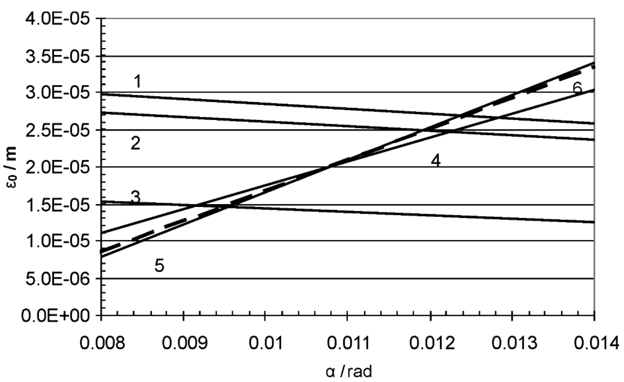

Figure 4: The method of linearisation (1, 2, 3) through the points ($0 : \varepsilon^1_0$) and ($a* : \varepsilon^*_0$) and of derivation in the point ($a* : \varepsilon^*_0$) (4, 5, 6) Standard (3, 4), guider (1, 5), Clone cylinder (2, 6)

Slika 4: Metoda linearizacije (1, 2, 3) skozi točke ($0 : \varepsilon^1_0$) in ($a* : \varepsilon^*_0$) in derivacije v točki ($a* : \varepsilon^*_0$) (4, 5, 6).
Standard (3, 4), vodilo (1, 5), valj klon (2, 6)

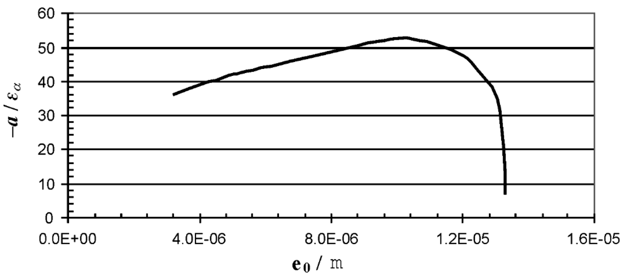

Figure 5: Relationship lubricant layer thickness – rolling gap geometrical shape

Slika 5: Odvisnost debeline plasti maziva od geometrične oblike valjalniške reže

$$\frac{\varepsilon_0^M}{\varepsilon_0^T} = (S\alpha^{1/2} + B)/\alpha + (C\alpha^{1/2} + L)/\alpha^2$$

$$Q = \alpha^{1/2} \qquad W = (\varepsilon_0^{\mathrm{M}} / \varepsilon_0^{\mathrm{T}})\alpha^2 \tag{9}$$

The results of the calculation are shown in **Table 4**.

Table 4: Some statistical data on the regression analysis
Tabela 4: Nekateri statistični podatki o regresijski analizi

S	0.8519
B	–0.2633
C	0.0346
L	–0.0015
R_{sq}	0.99999
F	66829.2
d. f.	5
Q, *W*	Abscissa and ordinate

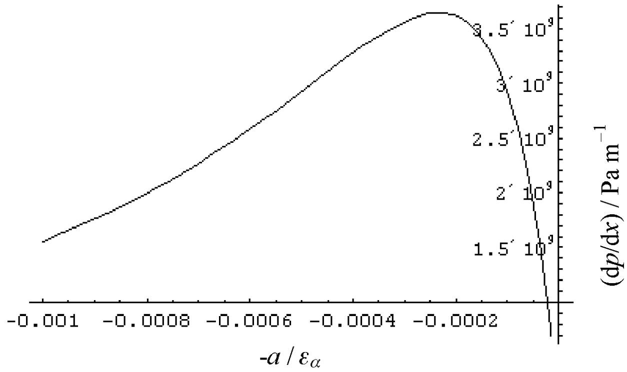

Figure 6: Change of pressure gradient for the gripping angler α = 0,02 rad and rolls diameter R = 0.2 m in dependence of the ratio (–*a*) / (ε_{-a}) in the range of – 0.001 m to – 0.000015 m and other parameters as for **Table 2**

Slika 6: Sprememba gradienta pritiska za kot oprijema α = 0,02 rad in premer valjev *R* = 0,2 m v odvisnosti od razmerja (–*a*)/(ε_{-a}) v območju od –0.001 m do – 0.000015 m in drugih pamaterov kot za **tabelo 2**

3 EFFECTS OF LUBRICANT INERTIA

The effect of lubricant inertia depends on the speed of the processing, the gap geometry and the rheological characteristics of the lubricant. In this work only investigations of the effect in the gap geometry is discussed. It was deduced with a numerical solution of the equation (1) applying the Monte-Carlo method with A = 1 965 512 m^{-1}, R = 0.2 m, ρ = 854 kg/m^3, dressing angle 0.02 rad and other data as in **Table 2** and it is shown in **Figure 4**.

The reliable analysis range lies both sides near of the point 2 in **Figure 5**. On the arc 2∩3 of the curve, by increasing of the strip lubricant layer thickness, the thickness of the lubricant layer in the initial section of the metal deformation zone is increased and the effect of inertia forces is decreased, while, on the arc 2∩1 of the curve the effect is opposite. For a selected processing, the vertex at the point 2 can be deduced with a regression analysis for determined processing parameters. In **Table 5** data on the effect of inertia forces on ε_0 calculated from equations (1a) and (2) using the Monte-Carlo method are given. The calculation was similar as for the curves in **Figure 2** and the data in **Table 2**. For the arc 2∩3 and dressing processing the correction for the lubricant layer thickness would be unnecessary, while, considering the inertia forces it is significant for the arc 2∩1. If the working velocity is increased from 16 m/s, as in **Table 5**, to 50 m/s for the angle 0,05 rad ε_0 = 13.854 · 10^{-6} m is deduced, while the consideration of inertia forces gives ε^{IN}_0 = 14,951 · 10^{-6} m.

Data in **Table 5** show that the shape of the gap in the area (–*a*, 0) prevails over the lubricant rheological cha-

Table 5: Effect of inertia forces(ε^{IN}_0) for dressing rolling
Tabela 5: Vpliv sil vztrajnosti (ε^{IN}_0) za oblikovalno valjanje

α/rad	0	0.011335	0.02	0.03	0.04	0.05
ε_0 /m	15.863 · 10^{-6}	12.255 · 10^{-6}	9.416 · 10^{-6}	7.244 · 10^{-6}	5.797 · 10^{-6}	4.695 · 10^{-6}
ε^{IN}_0 /m	15.863 · 10^{-6}	12.335 · 10^{-6}	9.542 · 10^{-6}	7.447 · 10^{-6}	5.959 · 10^{-6}	4.906 · 10^{-6}

Table 6: Dependence of the lubricant layer thickness on the gripping angle and the lubricant layer thickness on the strip ahead the deformation zone. Calculated using equations (1a) i (2).
Tabela 6: Odvisnost debeline plasti maziva od kota oprijema in debeline plasti maziva na traku pred zono deformacije. Izračunano z uporabo enačb (1a) in (2).

α/rad	ε_a/ m	ε_a/m	ε_a/m	ε_a/m	ε_a/m
	0.001	0.002	0.003	0,004	0.005
Without inertia	12.613· 10^{-6}	12.642· 10^{-6}	12.648· 10^{-6}	12.651· 10^{-6}	12.652· 10^{-6}
α = 0.01	12.793 · 10^{-6}	12.853 · 10^{-6}	12.884 · 10^{-6}	12.903 · 10^{-6}	12.912 · 10^{-6}
Without inertia	9.390 · 10^{-6}	9.407 · 10^{-6}	9.412 · 10^{-6}	9.414 · 10^{-6}	9.415 · 10^{-6}
α = 0.02	9.491 · 10^{-6}	9.539 · 10^{-6}	9.562 · 10^{-6}	9.578 · 10^{-6}	9.589 · 10^{-6}
Without inertia	7.225 · 10^{-6}	7.238 · 10^{-6}	7.241 · 10^{-6}	7.242 · 10^{-6}	7.243 · 10^{-6}
α = 0.03	7.286 · 10^{-6}	7.322 · 10^{-6}	7.340 · 10^{-6}	7.352 · 10^{-6}	7.361 · 10^{-6}
Without inertia	5.786 · 10^{-6}	5.793 · 10^{-6}	5.795 · 10^{-6}	5.796 · 10^{-6}	5.796 · 10^{-6}
α = 0.04	5.820 · 10^{-6}	5.848 · 10^{-6}	5.862 · 10^{-6}	5.871 · 10^{-6}	5.878 · 10^{-6}
Without inertia	4.788 · 10^{-6}	4.793 · 10^{-6}	4.794 · 10^{-6}	4.795 · 10^{-6}	4.795 · 10^{-6}
α = 0.05	4.809 · 10^{-6}	4.830 · 10^{-6}	4.841 · 10^{-6}	4.848 · 10^{-6}	4.854 · 10^{-6}

racteristics and the kinematics of the processing because the gripping angle approaches to zero. For this reason, the inertia forces do not affect significantly the lubricant behaviour by dressing processes, while these forces shold be considered by cold rolling. In **Table 6** the effect of ε_a (lubricant layer thickness on the sheet) on ε_0 (lubricant layer thickness at entrance cross section) is shown, as deduced using equation (2) and the Monte-Carlo calculation with R = 0.2 m and other data, as for **Table 5**. The value of ε_0 is increased for ≈1–2 %, while the value of ε_a increases for the sheet for five times for the rolling velocity up to 16 m/s.

In **Table 6** the effect of lubricant height on the sheet on the lubricant sheet on the entering section of the deformation zone is shown as function of lubricant inertia and gripping angle. The inertia effect increases significantly with the gripping angle and even faster with the lubricant height on the sheet ahead the rolls.

On **Figure 6** the change of pressure gradient ahead the rolling gap is shown with the maximum for x = –0.00025 m.

It is very difficult to obtain approximate analytical solutions of the differential equation (2). For this reason, it is analysed applying the numerical Monte-Carlo integration. The analysis should be complemented[13] considering surface roughness and with correlations with dependences to dynamkcal viscosity and rolling pressure.

4 CONCLUSIONS

On the base of the results of the calculations presented the following conclusions are proposed:

– The criteria of similarity are transferred acceptably with the solution of the equation (1a) from the standard over the guider to the cylinder clone applying the method of linearisation up to a gripping angle determined by linear programming. For a greater gripping angle the transfer of similarity criteria can be achieved using straight lines obtained with derivation in the point of singularity.
– The effect of inertia forces can be neglected for lower dressing speed, while this effect becomes significant for incresead speed of cold rolling.
– The use of the improved Mizunov-Grudev equation (8) gives solutions in good agreement with the numerical solution of equation (1) applying the Monte-Carlo method. Also, it is more practical for use than the transcendent equation (7).
– For a determined relation rolls diametere versus gripping angle very similar results are obtained using the Mizunov – Grudev relation and the transcendent equation (7).

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