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IDENTIFICATION OF MATERIAL PROPERTIES OF QUASI-UNIDIRECTIONAL CARBON-EPOXY COMPOSITE USING MODAL ANALYSIS

IDENTIFIKACIJA MATERIALNIH LASTNOSTI ZA KVAZIENOSMERNI KOMPOZIT OGLJIK-EPOKSI Z MODALNO ANALIZO

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This work focuses on the identification of material parameters of carbon-epoxy composite with continuous ultrahigh modulus fibers. The tested structure is a cantilever beam with a rectangular cross-section manufactured with forming several fiber bundles together, each wrapped with transverse layer of fibers. The wrapping fibers provide additional strength in transverse loading. The eigen-frequencies of the beam are experimentally assessed using piezoelectric transducers and laser sensor. Corresponding modal analysis is performed using finite element method and the axial Young's modulus is deduced with the mathematical optimization by minimizing the difference between measured and calculated eigen-frequencies. The model is then verified with transient analysis when the piezoelectric patches are excited by harmonic signals covering the first two eigen-frequencies.

Keywords: piezoelectric, composite, identification, finite element

Težišče dela je na identifikaciji materialnih parametrov za kompozit ogljik-epoksi iz neprekinjenih vlaken z ultravelikim modulom. Preizkušena struktura je konzolni nosilec s pravokotnim prerezom, izdelan s povezavo snopov vlaken ovitih s prečnimi plastmi vlaken, ki zagotavljajo dodatno trdnost pri prečni obremenitvi. Lastne frekvence nosilca smo eksperimentalno določili z uporabo piezoelektričnih transduktorjev in laserskih senzorjev. Modalno analizo smo izvršili z uporabo končnih elementov, osni Youngov modul pa je bil določen z matematično optimizacijo z minimaliziranjem razlike med izmerjenimi in izračunanimi lastnimi frekvencami. Model je bil nato preverjen s tranzientno analizo, pri kateri so bili piezoelektrični merilniki vzbujeni s harmonskim signalom, ki je obsegal dve prvi lastni frekvenci.

Ključne besede: piezoelektrik, kompozit, identifikacija, končni elementi

1 INTRODUCTION

Light-weight structures are nowadays necessary components in modern state-of-the-art products in all sorts of industries. These structures usually utilize the composite matererials in various form, such as shell-like laminates made from unidirectional layers or fabrics, wound or pultruded tubes and other profiles, or thick-walled components made by sandwiching composite skins and foam cores [1,2,3,4].

The increasing requirements on structural performance call for the use of embedded sensors and actuators and the construction of the so-called adaptive, smart or even intelligent structures that can respond to loading conditions in real time [10]. This enables, for instance, to monitor the condition of the structure [5], suppress vibrations or to adapt the desired shape [1,8], provided that proper electronic control circuits are applied.

Commonly used types of sensors and actuators are based on piezoelectric materials. The finite element modeling of the piezoelectric materials began with the first implementation in 1970 [2]. Many models have then been developed to simulate the piezoelectric effect, ranging from the simples using the similarity to the theory of thermo-elasticity to models [3], multi-purpose elements programmed for commercial software [11,12], up to complex models with full piezoelectric coupling incorporating layerwise approach for electric potential across layers [7] or quadratic variation of electric potential across the layer thickness [6].

The purpose of this work is to set up numerical model of composite beam with novel material structure and to use the piezoelectric actuators to identify the structural properties using experiment and the corresponding numerical simulation. This should be further extended for the application of monitoring the structural health in the future.

2 ANALOGY BETWEEN PIEZOELECTRICITY AND THERMAL EXPANSION

Let us consider the theory of piezoelectricity which assumes a symmetrical hexagonal piezoelectric structure and only the laminar piezoelectric effect (also called d_{31} effect, both direct and converse), i.e., the material is polarized in the thickness direction and the electric potential varies linearly across the thickness [9].

The classical stress-strain law (Hooke's law)

$$\boldsymbol{\sigma} = \boldsymbol{C}\boldsymbol{\varepsilon} \tag{1}$$

$\boldsymbol{\sigma}$ being the stress vector, $\boldsymbol{C}$ stress-strain matrix and $\boldsymbol{\varepsilon}$ the strain vector, is extended in this case by the piezoelectric coupling, hence

$$\begin{aligned} \boldsymbol{\sigma} &= \boldsymbol{C}\boldsymbol{\varepsilon} - \boldsymbol{e}\boldsymbol{E} \\ \boldsymbol{D} &= \boldsymbol{e}^T\boldsymbol{\varepsilon} + \in \boldsymbol{E} \end{aligned} \tag{2}$$

where $\boldsymbol{e}$ is the piezoelectric coupling matrix, $\boldsymbol{E}$ is the electric field vector, $\boldsymbol{D}$ is the vector of electric flux density (electric displacement), and $\in$ is the dielectric permittivity matrix.

In many applications, the electric potential can be considered as known (the piezoelectric material is in the actuator mode) and, therefore, the second equation in (2) does not need to be solved for the electro-mechanical behavior. This allows to model the problem of piezoelectricity using the analogy with thermal expansion [3]. This can prove very helpful if the used software does not contain piezoelectric features.

The stress-strain law with thermal expansion for one-dimensional problem can be written as

$$\sigma = E(\varepsilon - \alpha\Delta T) \tag{3}$$

where E is the Young's modulus, α the coefficient of thermal expansion and ΔT the change in temperature. The corresponding piezoelectric equation is

$$\sigma = E\varepsilon - e\frac{U}{d} \tag{4}$$

with U being the voltage across electrodes and d the distance between the electrodes. The analogy is obvious and it is possible to write directly the resemblance between

$$\alpha \approx \frac{e}{E} \quad \text{and} \quad \Delta T \approx \frac{U}{d} \tag{5}$$

3 EXPERIMENT

Experimental investigation of oscillations caused by harmonic excitations of quasi-unidirectional composite beam was carried out. The beam consists of unidirectional (0 degree) PITCH carbon fiber (K63712) sections each wrapped by 90 degree fibers and then stacked altogether. The matrix is composed of epoxy anhydride resin. The dimensions of the cross-section (see **Figure 1**) were (30 × 20) mm and the density was assessed to be ρ = 1838 kg/m^3. The beam was made by CompoTech company.

Two collocated piezoelectric patches DuraAct P876.A12 (see **Figure 2**) were glued with HBM Z70 glue to the beam upper and lower surfaces. The dimensions of the patch are (61 × 35 × 0.5) mm while, the size of the active piezoelectric material (E = 61.8 GPa, e = 5.6 C/m^2, ρ = 7760 kg/m^3), which is enclosed in a protective foil (E = 8.2 GPa, ρ = 1528 kg/m^3), was of only (50 × 30 × 0.2) mm.

Figure 1: The cross-section of the composite beam showing the wrapped unidirectional sections

Slika 1: Prečni prerez kompozitnega nosilca, ki prikazuje ovite osne dele

Figure 2: Top and bottom view of the DuraAct P876.A12 piezoelectric transducers (patches)

Slika 2: Pogled od zgoraj in od spodaj piezoelektričnega transduktorja DuraAct P876. A12

The beam of the free length of 1312 mm was clamped at one end so that the gap between the fixture and the active piezoelectric area was 10 mm. The piezoelectric patches were loaded by a sine signal from the generator (± 2V) connected to voltage multiplier (50 times) with final amplitude of 100 V. The connection of patches resulted in bending of the beam. Laser sensor OptoNCDT was used to measure the deflections of the free end. The scheme of the experimental setup is shown in **Figure 3**.

The two lowest bending eigen-frequencies of the structure were found by sweeping the generator fre-

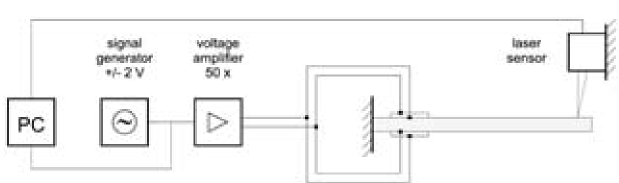

Figure 3: Scheme of experimental setup

Slika 3: Shema eksperimentalne postavitve quency and searching for the largest steady oscillations. The values are f_1 = 21 Hz and f_2 = 134 Hz. The latter analysis investigated the response to frequencies around the two eigen-frequencies, namely the intervals $\langle 15, 25\rangle$ Hz and $\langle 125, 140\rangle$ Hz. The amplitudes of the steady oscillations A were measured for each frequency using the laser sensor.

Numerical analysis

Finite element model of the investigated structure was designed in MSC.Marc/Mentat software utilizing the analogy between piezoelectricity and thermal expansion as introduced in (5). The beam consisted of eight-node solid elements (with assumed strain option) as shown in **Figure 4**. The prescribed boundary conditions for the simulation of the clamped part are shown in **Figure 4**, also. The detail of how the materials were modeled within the structure is obvious from the cross-section displayed in **Figure 5**.

The elasticity properties of the new material structure were unknown. Since only the bending behavior was of interest, it was possible to assume the composite and piezoelectric materials to be homogeneous and isotropic (the elasticity constants correspond to axial components, Poisson's ratios ν = 0.3). The goal in this work was therefore to identify at least the Young's modulus from the comparison of experiment and numerical model. Simple optimization loop with interval partitioning was used to minimize the error

$$\Delta = \sum_{i=1}^{2} \left\{ \left(f_i^{\mathrm{EXP}} - f_i^{\mathrm{FEA}} \right)^2 \right\} \qquad (6)$$

where f_i^{EXP} and f_i^{FEA} are the measured and calculated eigen-frequencies, respectively. The optimal value of Young's modulus was found to be 245 GPa whereas the corresponding frequencies differed by less than 1 %.

The following investigation focused on the comparison of the measured frequency characteristics of the composite beam with the results of the numerical model. The piezoelectric material was excited by harmonic signals covering similar spectra as the experimental. The calculated and measured amplitudes of steady oscillations A are shown in **Figure 6**. The decrease in eigen-frequencies (given by the peaks in the graphs) from those obtained by modal analysis (approx. by 3 %) and in the maximum amplitudes can be explained using of the single-step Houbolt time integration scheme in MSC.Marc. The time step was set to 1/50 of the corresponding signal period.

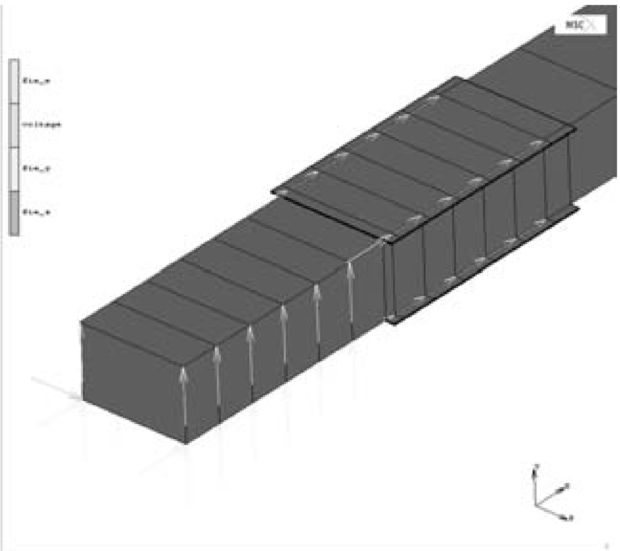

Figure 4: Finite element model of beam with attached piezoelectric patch and applied boundary conditions
Slika 4: Končnoelementni model nosilca s pritrjenimi piezoelektričnimi merilniki in uporabljene mejne vrednosti

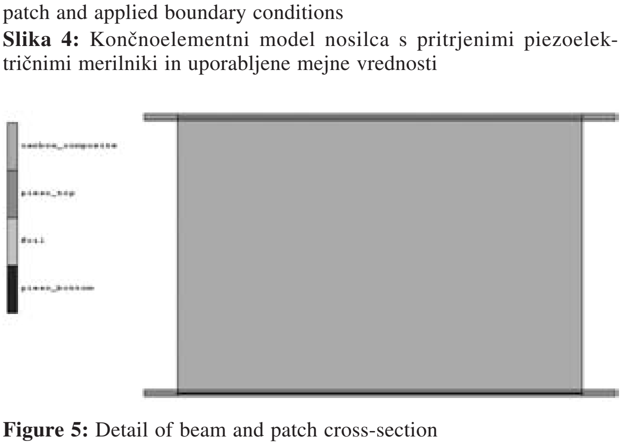

Figure 5: Detail of beam and patch cross-section
Slika 5: Detajl prečnega prereza nosilca in merilnika

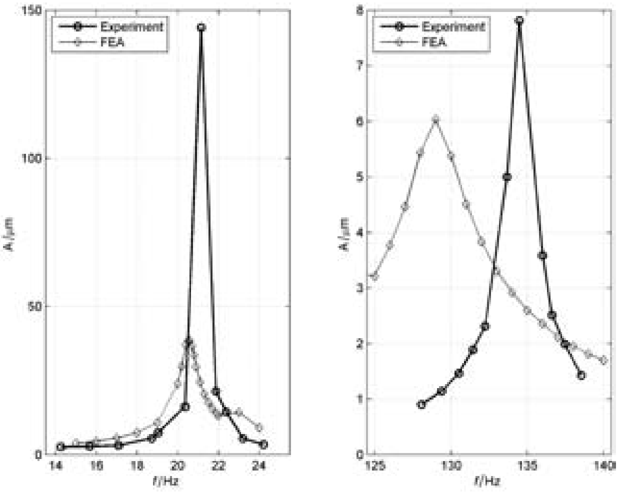

Figure 6: Amplitude characteristics around first two eigen-frequencies
Slika 6: Značilnosti amplitud okoli prvih dveh lastnih frekvenc

4 CONCLUSIONS

The experimental analysis of frequency characteristics of hybrid composite cantilever beam was carried out. The beam consists of carbon-fibers and epoxy anhydride resin. Two collocated piezoelectric patches glued to its surface were applied to induce bending.

Corresponding finite element model was designed in MSC.Marc/Mentat software using the analogy between piezoelectricity and thermal expansion. The axial modulus of the quasi-unidirectional material was identified by modal analysis using simple optimization loop with interval partitioning. Moreover, the comparison of the calculated and measured frequency characteristics when the piezoelectric actuators were excited with harmonic signals around the first two eigen-frequencies was performed.

Acknowledgements

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