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FATIGUE-CRACK PROPAGATION NEAR A THRESHOLD REGION IN THE FRAMEWORK OF TWO-PARAMETER FRACTURE MECHANICS

DVOPARAMETRSKA LOMNO MEHANSKA ANALIZA HITROSTI UTRUJENOSTNE RAZPOKE BLIZU PRAGA PROPAGACIJE

Stanislav Seitl, Pavel Hutař

Institute of Physics of Materials, Academy of Sciences of the Czech Republic, Zizkova 22, 616 62 Brno, Czech Republic
seitl@ipm.cz

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A two-parameter constraint-based fracture mechanics method was applied to the problems of the fatigue-crack propagation rate near a threshold region. Two geometries of specimens with different values of constraint were analyzed. The experimentally obtained results were compared with numerical data. The presented result makes it possible to relate experimentally measured data obtained from specimens with different geometries, and thus contributes to more reliable estimates of the residual fatigue life of a structure.

Key words: constraint, *T*-stress, threshold values, fatigue crack growth rate, high cyclic fatigue,

Dvoparametrska vpetostna metoda je uporabljena pri problemu širjenja utrujenostne razpoke blizu praga propagacije. Analizirani sta dve geometriji preizkušanca z različno vpetostjo. Eksperimentalni podatki so primerjani z numeričnimi. Rezultati omogočajo, da se poveča eksperimentalne izsledke s preizkušancev z različno geometrijo in omogočajo bolj zanesljivo oceno preostale trajnostne dobe neke strukture.

Ključne besede: vpetost, vrednosti praga, hitrost rasti utrujenostne razpoke, visokociklična utrujenost

1 INTRODUCTION

The understanding of constraint effects on the fatigue-crack propagation rate (FCPR) is relevant for the prediction of the operating life of engineering structures. The elastic *T*-stress, the second term of Williams's series expansion[1] for linear elastic crack-tip fields, represents the stress acting parallel to the crack plane. In paper[2] results relating to FCPR measurements for a 2024 aluminum alloy of two different specimen geometries are investigated. The FCPR was found to be sensitive to the specimen's geometry in the Paris region of the d*a*/d*N*–*K* curve as well as in the threshold values. The same trends are quantified for different levels of constraint in the vicinity of the fatigue-crack tip by means of the *T*-stress. This has been published for the ratio $R \approx 0$ in paper[3]. These findings are contradicted by the results published in [4]. The experiments are focused on the fatigue-crack growth data from three different specimen geometries obtained for nickel-based super-alloys and mild steel. The recently published study[5] describes experiments on edge-bending specimens SE(B) and C(T) specimens to characterize the fatigue-crack growth rates of Inconel 718, and in this study no difference in the crack-growth rate between the two kinds of specimens was observed.

Within two-parameter fracture mechanics (2PFM) the stress field near the crack tip is expressed by means of two parameters, i.e., the stress-intensity factor, *K*, and the *T*-stress. The stress field at the crack tip can then be expressed for a normal mode (mode I) of loading as

$$\sigma_{ij} = \frac{K_1}{\sqrt{2\pi r}} f_{ij}(\theta) + T\delta_{1i}\delta_{1j} \qquad (1)$$

where $f_{ij}(\theta)$ is a known function of the polar angle θ, and δ_{ij} and δ_{1j} are the Cronecker deltas, taking the value 1 if i and j are equal, and 1 and 0 otherwise.

The work reported here uses 2PFM for a description of the different threshold values of a FCPR to predict the effect of constraint on the retardation or acceleration of the crack growth in high cyclic fatigue loading. The corresponding calculations are performed according to the finite-element method (FEM). The experimental measurements of FCPR on two kinds of specimens with a different geometries were compared and so were the experimental data covering the effect of constraint.

2 TESTS OF THE FATIGUE-CRACK PROPAGATION RATE NEAR TRESHOLD VALUES

The effect of constraint on the fatigue-crack growth rate near the threshold values was experimentally studied in relation to two different specimens: the M(T) (the middle-tension specimen) and C(T) (the compact-tension specimen). The M(T) specimen produced a low level of constraint (negative values of the *T*-stress) and the C(T) specimen produced a high level of constraint (positive

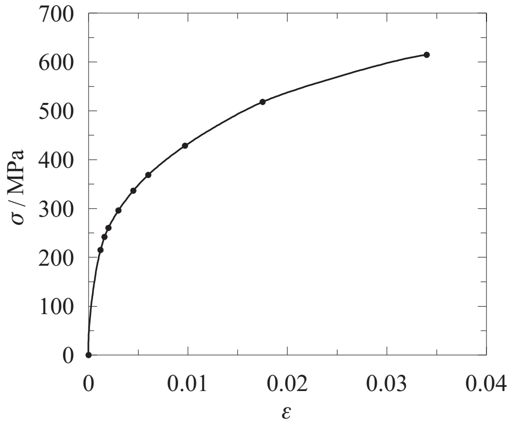

Figure 1: Cyclic stress-strain curve of the material used for experiments
Slika 1: Ciklična napetost-deformacija odvisnost materiala, ki je bil uporabljen za preizkuse

values of the T-stress) e.g., [6]. The thickness of both types of specimens was the same, B = 10 mm. The experimental results were obtained for steel with 0.45 % C. The material properties corresponding to the cyclic strain-stress curve (**Figure 1**) are: Young's modulus E = $2.1 \cdot 10^5$ MPa, Poisson's ratio ν = 0.3, cyclic yield stress σ_0 = 350 MPa and hardening exponent n = 0.314. All the experiments were performed under the same conditions: room temperature and a loading stress ratio $R \approx 0$ (close to a pulsating cycle R = 0.1). The crack length was measured optically with a resolution of 0.01 mm. The experimental data were evaluated according to ASTM E647[7]. The threshold values were determined at crack-growth rates of 10^{-8} mm/cycle.

The following results were obtained, see **Figure 2**. The fatigue-crack growth rate was found to be higher for the M(T) specimen with a negative value of T-stress (low constraint) than for the C(T) specimen with a positive value of T-stress (high constraint), when subjected to the same nominal range of the stress-intensity

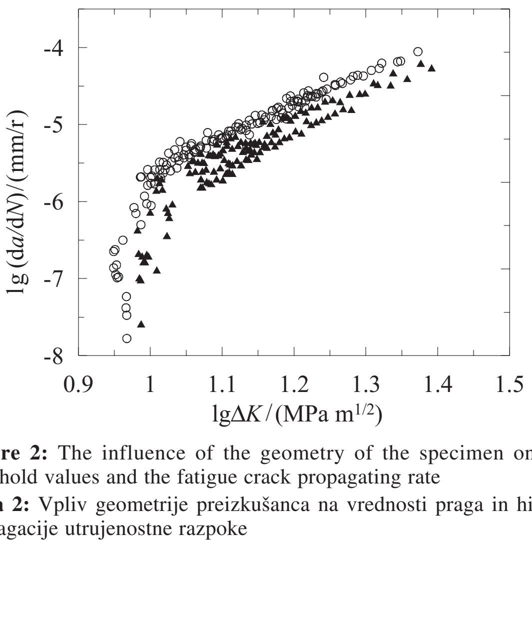

Figure 2: The influence of the geometry of the specimen on the threshold values and the fatigue crack propagating rate
Slika 2: Vpliv geometrije preizkušanca na vrednosti praga in hitrost propagacije utrujenostne razpoke

factor, ΔK. Significant differences were seen between the threshold values of the FCPR. The corresponding parameters of the Paris-Erdogan law are then: $C = 6.607 \cdot 10^{-11}$, m = 4.609 and K_{th} = 8.81 MPa m$^{1/2}$ for the M(T) specimen and $C = 2.883 \cdot 10^{-10}$, m = 3.662 and K_{th} = 9.66 MPa m$^{1/2}$ for the C(T) specimen.

3 CONSTRAINT-BASED DESCRIPTION OF A FATIGUE CRACK

The plastic-zone size around the crack tip depends upon many variables, such as the yield stress, the applied stress, the specimen geometry and the crack geometry. According to one-parameter linear-elastic fracture mechanics (LEFM), there is a single-valued relation between the size of the plastic zone and the corresponding value of the stress-intensity factor, K_I, which controls the FCPR (e.g.,[8]). Following this, one of the parameters, say $R_p = R_p(K)$, which characterizes the size of the plastic zone, can be used as the controlling variable, and the Paris-Erdogan law[9] $da/dN = C(K)^m$ can be rewritten in the form

$$\frac{\mathrm{d}a}{\mathrm{d}N} = F\left[R_\mathrm{p}(K)\right] \tag{2}$$

Therefore, the size of the plastic zone can be considered as a potential parameter with a direct physical meaning to describe the FCGR. Based on the assumptions of the 2PFM, the size of the plastic zone depends on the given value of K and the T-stress (the level of constraint), $R_p^* = R_p^*(K,T)$, and Equation (2) takes the form:

$$\frac{\mathrm{d}a}{\mathrm{d}N} = F\left[R_\mathrm{p}^*(K)\right] \tag{3}$$

Equation (3) considers the effect of the stress field at the crack tip on the FCPR, which included the non-singular term (T-stress), and this Equation (3) can be used to quantify the effect of the constraint.

In the present study, see e.g.[3], the plastic-zone area, $S_p(K,T)$, is used as a parameter controlling the FCPR, and it is assumed that the crack will propagate at the same fatigue rate if the values of $S_p(K,T)$ (corresponding to different combinations of the K and T parameters) are the same.

Let us take the level of constraint corresponding to the zero value of the T-stress as a reference state and denote the corresponding area of the plastic zone as $S_p^0 = S_p(K,T = 0)$, based on the assumption of LEFM it is

$$S_\mathrm{p}^0 = S_\mathrm{p}(K,T=0) = \psi(\nu,T=0)(K/\sigma_0)^4 \tag{4}$$

where $\psi(\nu,T = 0)$ is a function of Poisson's ratio, ν, and the cyclic yield stress, σ_0 of the material used. The function can be determined conventionally by substituting the singular terms of the elastic stress field into the von Mises' yield criterion or numerically using the FEM. Similarly, let us denote the area of the plastic zone around the crack tip corresponding to a structure with a non-zero level of constraint as $S_p = S_p(K,T \neq 0)$, and express it in the form

$$S_p(K,T \neq 0) = \psi(\nu,T \neq 0)(K/\sigma_0)^4 \qquad (5)$$

Both quantities, $\psi(\nu,T = 0)$ and $\psi(\nu,T \neq 0)$, were calculated numerically by using a modified boundary-layer analysis (MBLA). The calculations of the plastic-zone size and area were performed for different values of the stress-intensity factor as a function of the ratio of the T-stress and the yield stress, T/σ_0, in an interval of a real ratio < –0.8; 0.4 > (see also[10]). The chosen values of the K factor and the ratio T/σ_0 correspond to the applied levels of the external stress and the constraint in the presented experiments. The dependence obtained can then be expressed in the form

$$\lambda\left(\frac{T}{\sigma_0}\right) = 1 - 0.33\left(\frac{T}{\sigma_0}\right) + 0.66\left(\frac{T}{\sigma_0}\right)^2 - 0.445\left(\frac{T}{\sigma_0}\right)^3,$$

see **Figure 3**.

The relation between the plastic-zone sizes with and without using the T-stress was found in the form:

$$S_p(K,T) = \lambda^4(T/\sigma_0)S_p^0 \qquad (6)$$

Let us further define the effective value of the stress-intensity factor, K^{eff}, using the equality (6) that relates the plastic-zone area for $T = 0$ and $T \neq 0$ in the form:

$$K^{eff}(T) = \lambda(T/\sigma_0)K(T = 0) \qquad (7)$$

Where the application of the $K^{eff}(T)$ for the description of the fatigue-crack growth rate takes the level of the applied stress and the constraint into account, the Paris-Erdogan equation can be rewritten in the form

$$da/dN = C[\lambda(T/\sigma_0)K]^m \qquad (8)$$

Equation (8) represents a modified form of the Paris-Erdogan law and makes it possible to account for the effect of the constraint on the fatigue-crack growth rate. C and m are material constants obtained for conditions corresponding to $T = 0$. The value of the T-stress represents the level of constraint corresponding to the given specimen geometry.

4 DISCUSSION

The approach presented is a phenomenological one and does not consider the effect of plasticity-induced crack closure; it proceeds from the assumption of LEFM. The two-parameter fracture methodology is based upon the assumption that the fracture behaviour of two different bodies is the same if both encompass the same value of the stress-intensity factor, K, and the same range of the constraint parameter T-stress. Note that the positive values of the T-stress correspond to a high constraint level and negative values to a low constraint. The two-parameter description characterized the stress state near the crack tip more accurately, and can explain the effect of the outer geometry of the structure. On the other hand, this approach does not take account of the corresponding mechanism of the fatigue-crack growth rate or the microstructure of the material.

The correlation of the dependence $da/dN - K$ related to a zero constraint level for the steel specimens is shown in **Figure 4**. The corresponding Paris-Erdogan law parameters are $C = 1.473 \cdot 10^{-10}$, $m = 4.013$ and $K_{th} = 9.44$ MPa m$^{1/2}$. The points corresponding to the lower level of constraint (M(T) specimen) are shifted to lower values of the FCPR. Similarly, the points corresponding to a higher level of constraint (the C(T) specimen) are transformed into higher values of the FCPR. The scatter of the experimental results after this transformation is smaller in comparison to the data in **Figure 2**. Finally, it is possible to make an approximation of all experimental data by the one material curve, which is independent of the outer geometry of the specimen, with reasonable experimental scatter corresponding to a zero value of the constraint.

Similarly, the influence of the constraint can be described in the threshold region, see **Figure 4**.

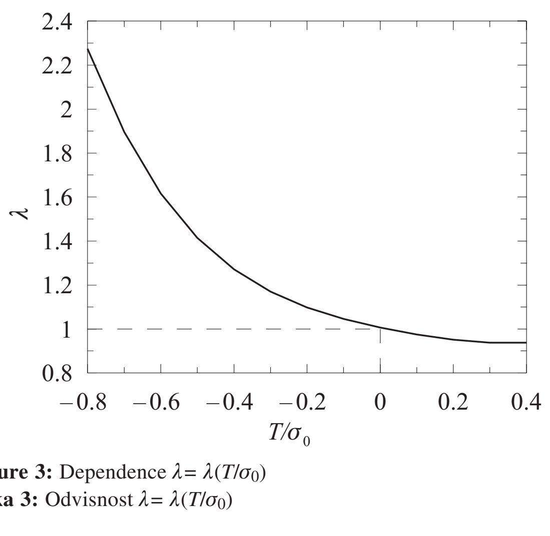

Figure 3: Dependence $\lambda = \lambda(T/\sigma_0)$
Slika 3: Odvisnost $\lambda = \lambda(T/\sigma_0)$

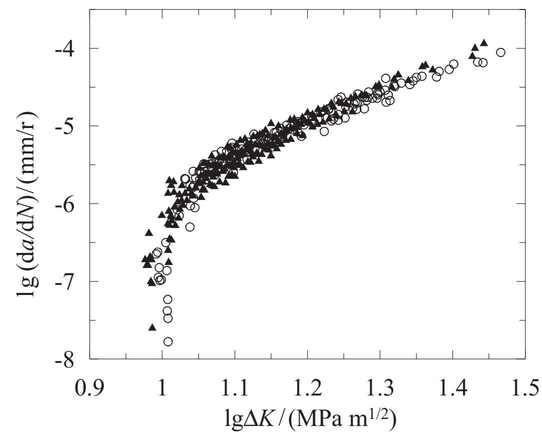

Figure 4: The experimental data from **Figure 2** correlated to a zero value of the constraint
Slika 4: Eksperimentalne vrednosti s **slike 2** korelirane na nično vrednost vpetosti

Following these observations it can be concluded that the fatigue crack will not propagate if the threshold value for zero *T*-stress is smaller than the effective range of the stress-intensity factor.

$$K^{\mathrm{eff}}(K,T) < K_{\mathrm{th}}(T = 0) \qquad (9)$$

5 CONCLUSIONS

This work describes the constraint effect for fatigue-crack propagation near the threshold values in the context of the requirement for improved life-prediction methods. A combined experimental and modeling approach is used. Experimental measurements of the FCPR were made on two kinds of specimens with different shapes: i.e., M(T) (low level of constraint) and C(T) (high level of constraint) specimens.

The plastic zone is considered as a controlling variable for near-threshold fatigue-crack behaviour and a simple procedure makes it possible to estimate the changes of the fatigue-crack propagation rate in the threshold region due to different constraint levels formulated quantitatively.

The effective stress-intensity factor, K^{eff}, becomes smaller ($T > 0$) or larger ($T < 0$) than the nominally applied one, and hence the crack propagation is slower or faster, and the threshold values are lower or higher, *than would be predicted without any knowledge of the constraint effect*.

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