UDK 539.42:669.71:519.6
Izvirni znanstveni članek

ISSN 1580-2949
MTAEC9, 39(3)83(2005)

NUMERICAL DETERMINATION OF *J-R* CURVE USING VOID MODEL

NUMERIČNA DOLOČITEV ODVISNOSTI *J-R* NA PODLAGI MODELA MIKROVOTLIN

Vladislav Laš, Jan Očenášek, Vlastimil Vacek

Department of Mechanics, Faculty of Applied Sciences, University of West Bohemia, Univerzitní 8, 306 14 Pilsen, Czech Republic
las@kme.zcu.cz

Prejem rokopisa – received: 2004-09-17; sprejem za objavo – accepted for publication: 2005-05-09

The paper presents the determination of the *J-R* curve for an aluminium alloy. *J*-integral value is determined by numerical simulation of the test prescribed in ASTM E 1820-01. The crack growth Δa corresponding to the load increment is determined numerically. The Complete Gurson Model (CGM) based upon void models is adopted as a constitutive relation. This model describes the nucleation, growth and the coalescence of voids and it considers the constraint effect as well. The advantage of this approach is when compared with the standard experimental determination of the *J-R* curve that the number of experiments is reasonably lower in this case since only the experiments for the determination of the CGM parameters are needed. These experiments require much less effort and are less time demanding. Numerical simulations for the determination of the *J-R* curve were performed for the CT specimen. The finite element method (FEM) code ABAQUS was used for the computation. A subroutine including the CGM was implemented into the code. Static loading and room temperature were assumed in all simulations.

Key words: aluminium, *J-R* curve, numerical simulation, Complete Gurson Model, CT specimen

Predstavljena je določitev odvisnosti *J-R* za aluminijevo zlitino. J-integral je bil določen z numerično simulacijo preizkusa po ASTM E 1820-01. Napredovanje razpoke Δa pri povečanju obremenitve je bilo določeno numerično. Kot konstitutivna odvisnost je uporabljen Popoln Gursonov model (CGM) na podlagi propagacije mikrovotlin. Ta model opisuje nastanek, rast in koalescenco mikrovotlin ter upošteva tudi vpliv vpetosti. Prednost tega približka v primerjavi s standardno eksperimentalno določitvijo odvisnosti *J-R* je pomembno manjše število preizkusov, ker so za CGM potrebni le enostavni preizkusi. Numerična simulacija je bila izvršena za CT-preizkušanec. Za izračun je uporabljena metoda končnih elementov (FEM) in program Abaqus, v katerega je vstavljena podrutina s CGM. Pri vseh preizkusih smo uporabljali statično obremenjevanje pri sobni temperaturi.

Ključne besede: aluminij, krivulja *J-R*, numerična simulacija, popoln Gursonov model (CGM), CT-preizkušanec

1 INTRODUCTION

The determination of the *J-R* curves based upon the ASTM E 1820-01 standard [2] is rather time consuming and requires number of experiments. A minimum of 15 specimens must be tested in order to obtain a reliable *J-R* curve. Effort has been made to obtain the *J-R* curves by a less complicated experiment combined with numerical simulations. One of these methods is the Single-sample *J*-integral Test. For this fracture testing the on-line or continuous crack monitoring is required. This is generally performed by the Unloading Compliance Measurement (ASTM E 813-87) or by Electric Potential Drop methods. Another possibility for the determination of the *J-R* curve is the Load Normalization Technique. This technique does not require the on-line monitoring and it is based on the principle of load separation. The load may be mathematically expressed as a function of the crack length and the plastic deformation. The evaluation procedure uses the load-line displacement diagrams (see e.g. [5]). Another method used for the determination of the *J-R* curve is the Direct Method [1], which employs the load-load point displacement records of a single specimen test in the procedure. Then, the Direct Method Program, including necessary input data, calibration functions and elastic-plastic analysis, is applied. The advantages of this procedure are that it avoids the accuracy problems when measuring the crack growth, and the reduction of the testing time. Some results are presented e.g. in [1]. The possibility to determine the *J-R* curve without testing the standard specimens is discussed in this paper. Such approach requires only simple tensile tests and proper numerical simulations.

2 EXPERIMENTAL DETERMINATION OF *J-R* CURVE

The *J-R* curve can be determined experimentally according to the ASTM standard [2]. This procedure requires large number of experiments fulfilling the assumptions for the specimens used and for the measurement procedures. Three types of specimens are used: single edge bend specimen, compact specimen and disc-shaped compact specimen with fatigue pre-crack. The *J-R* curve determination procedure consists of loading the specimen to a given level and then determining the *J*-integral value and the crack length increment Δa. Thus, one point of the *J-R* curve is obtained. The minimum number of points to be determined is 15, whereas each point must be obtained from measurement on a new specimen subjected to higher load level, thus exhibiting longer crack length. In order to measure the crack length, each specimen must be broken using brittle fracture after cooling.

Since the numerical determination of the *J-R* curve in this work is performed by the simulation of experiment, the basic relations for the *J*-integral determination and the procedure of the relevant crack length increment Δa_x determination necessary for the experiment evaluation are presented below. CT specimen is used.

For the *J*-integral determination so-called load-line displacement curve (*P* versus *v*) is used. A typical curve is shown in **Figure 1a**, where A_{pl} represents the area corresponding to plastic work. The *J*-integral is then determined as:

$$J = \frac{K^2(1-\nu^2)}{E} + J_{pl} \tag{1}$$

where the first part of equation (1) represents the elastic value of *J*-integral, *K* is the stress intensity factor, ν is Poisson's ratio, *E* is Young's modulus, and J_{pl} represents the *J*-integral plastic component. The stress intensity factor is determined for the relevant force P_i as:

$$K_i = \frac{P_i}{(BB_N W)^{1/2}} f\left(\frac{a_i}{W}\right) \tag{2}$$

where $f(a_i/W)$ is the compliance function, a_i is the crack length, *W* is the specimen width, *B* is the specimen thickness and B_N is the specimen thickness measured at side grooves. The *J*-integral plastic component is determined from the loading-force plastic work as:

$$J_{pl(i)} = \left[J_{pl(i)} + \left(\frac{\eta_{(i-1)}}{b_{(i-1)}}\right)\frac{A_{pl(i)} - A_{pl(i-1)}}{B_N}\right]\left[1 - \gamma_{(i)}\frac{a_{(i)} - a_{(i-1)}}{b_{(i)}}\right] \tag{3}$$

where

$$\eta_{(i-1)} = 2.0 + 0.522 b_{(i-1)}/W$$
$$\gamma_{(i-1)} = 1.0 + 0.76 b_{(i-1)}/W \tag{4}$$

The crack length increment is measured optically. After loading to the given level the specimen is unloaded and the free crack surfaces are heat tinted. Then the

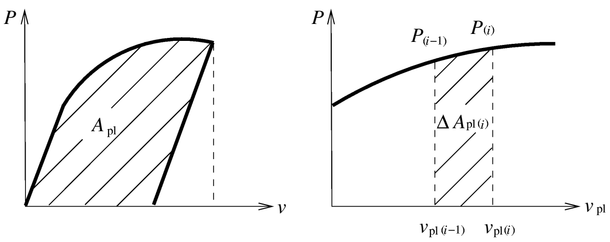

Figure 1: a) Load – line displacement curve (*P* versus *v*), b) Plastic work $\Delta A_{pl(i)}$ increment

Slika 1: a) Odvisnost obremenitev-premik obremenitvene črte (*P* v odvisnosti od *v*); b) Poveček plastičnega dela $\Delta A_{pl(i)}$

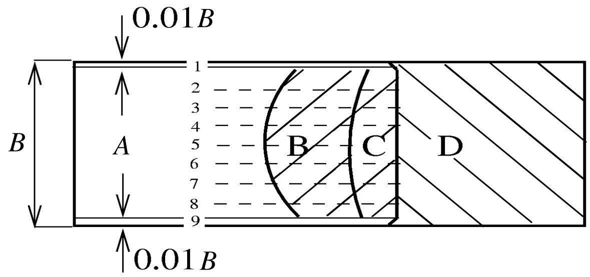

Figure 2: Fracture surfaces (A – brittle breaking surface, B – crack increment, C – pre-cracked surface, D – notch surface) and 9 points for the crack length increment measurement

Slika 2: Površine preloma (A – površina krhkega preloma, B – poveček propagacije razpoke, C – začetna površina razpoke, D – površina zareze) in 9 točk meritev povečka dolžine razpoke

specimen is cooled and brittle broken. **Figure 2** shows the surfaces of the CT specimen after brittle fracture and 9 locations for the crack increment measurement.

The crack length increment value is determined as:

$$\Delta a = \frac{1}{8}\left(\frac{\Delta a_1 + \Delta a_9}{2} + \sum_{i=2}^{8}\Delta a_i\right) \tag{5}$$

where $\Delta a_i = a_i - a_0$. The corresponding values of *J* and Δa are recorded into the diagram together with the construction lines, offset line and two exclusion lines, which define points from which the *J-R* curve will be determined (see **Figure 3**). The way of drawing the particular lines is given by the standard [2]. The *J-R* curve is then the power law regression line:

$$\ln J = \ln C_1 + C_2 \ln\left(\frac{\Delta a}{k}\right) \tag{6}$$

determined from points between two exclusion lines, where $k = 1$ mm (or $k = 0.0394$ in). Constants C_1 and C_2 are determined with the regression analysis. Value J_Q is given by the *J-R* curve and the offset line crossing point, Δa_{min} and Δa_{limit} are given by the *J-R* curve and the exclusion lines crossing points. The values Δa_{min} and Δa_{limit} and the exclusion lines then determine the feasible domain of the *J* values.

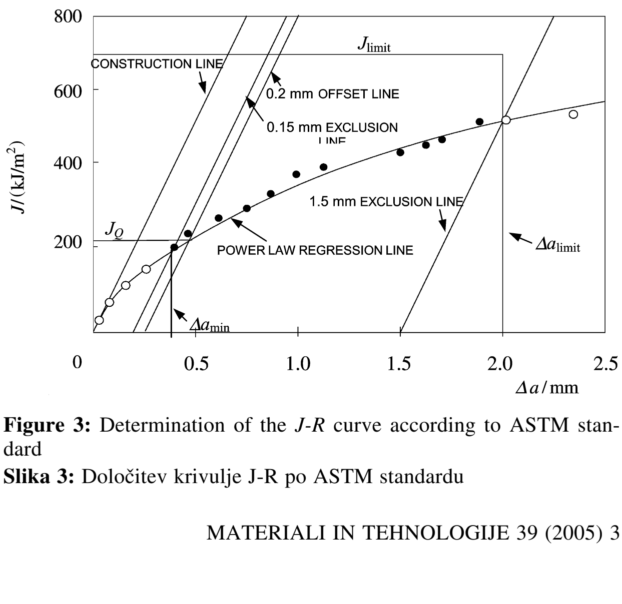

Figure 3: Determination of the *J-R* curve according to ASTM standard

Slika 3: Določitev krivulje J-R po ASTM standardu

3 COMPUTATIONAL MODEL

Since the experimental determination of the *J-R* curve is not simple, other possibilities for the determination of this curve are sought. One of these possibilities is a numerical simulation of the test prescribed in ASTM standard [2]. A proper model describing the real behaviour of the material during damage (the crack propagation) is necessary in order to obtain a reliable *J-R* curve. Since the stress-strain field near the crack tip cannot be described by common constitutive relations the material model used must include e.g. the effect of triaxiality, the nucleation, growth and coalescence of voids existing in the zone with high stress concentration. The above requirements are fulfilled by certain so-called void models, where the dilatation plasticity is used. This means that the plasticity condition is also function of the mean normal stress and that the volume change takes place, which is not considered in classical plasticity models.

One of the real plasticity models is the Complete Gurson Model (CGM), which eliminates the disadvantages of the Gurson Model (GM) and also of the modified Gurson Model (GTN) implemented by Tvergaard and Needleman [13].

The CGM yield function has the same form as that of GTN model:

$$\phi(\sigma_e, \sigma_k^k, \sigma_M, f^*) = \\ = \frac{\sigma_e}{\sigma_M} + 2q_1 f^* \cos h\left(\frac{q_2 \sigma_k^k}{2\sigma_M}\right) - 1 - (q_1 f^*) = 0 \quad (7)$$

where σ_e is the macroscopic von Misses equivalent stress, σ_M is the actual yield stress of the matrix material, σ_k^k is the trace of the macroscopic Cauchy stress tensor, f^* is the modified void volume fraction, q_1, q_2 are constants introduced by Tvergaard. This model involves phases of nucleation, growth and coalescence of voids [13]. Contrary to the GTN model, for which 6 parameters must be determined experimentally, only 4 parameters (f_0 initial void volume fraction, ε_N mean nucleation equivalent plastic strain, s standard deviation and f_N volume fraction of void nucleating particles) are to be determined. It is particularly important that it is not necessary to determine experimentally the critical void volume fracture f_C, which was considered to be a material constant in GTN and, as a matter of fact, it depends also on the stress state. Zhang eliminated this disadvantage [15] by the introduction of the Thomason plastic limit load model into GTN and by this way the CGM was created. Parameters f_0, ε_N, s and f_N are determined by numerical fitting to the experimental data, where for the notched bar subjected to tension the relation load P versus notch diameter reduction Δd is recorded. Other CGM parameters are assumed to be q_1 = 1.5, q_2 = 1.

CGM enables the numerical simulation of the crack growth and the determination of the crack length increment Δa as well as the simulation of the load-line displacement curve P versus v (**Figure 1**), from which the relevant value of the J-integral can be determined using relations in equation (1) to equation (4).

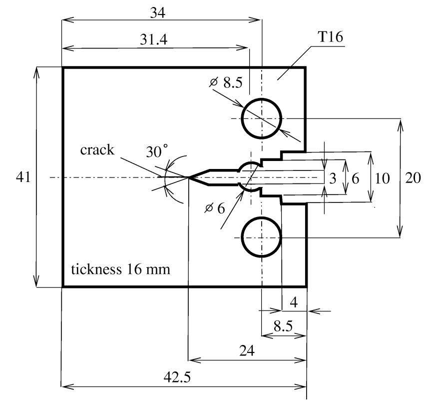

Figure 4: CT specimen dimensions
Slika 4: Dimenzije CT-preizkušanca

4 *J-R* CURVE NUMERICAL SIMULATION

The problem was solved using the finite element method programme ABAQUS, into which the CGM was implemented as a subroutine [15]. Static loading and room temperature were assumed in the analysis. The tested material was aluminium alloy with material constants:

Young's modulus E = 6.9122·10^4 MPa, Poisson's ratio ν = 0.315, yield stress σ_y = 280 MPa. The true stress-strain diagram was available, too. For the CGM the following parameter values were used: q_1 = 1.5, q_2 = 1, f_0 = 0.02, ε_N = 0.1, s = 0.1, f_N = 0.01. As stated above, these parameters were obtained by the numerical simulation of a tensile test of a notched bar of circular cross-section. Numerical fitting to the experimental data of the loading force P versus diameter reduction Δd was carried out.

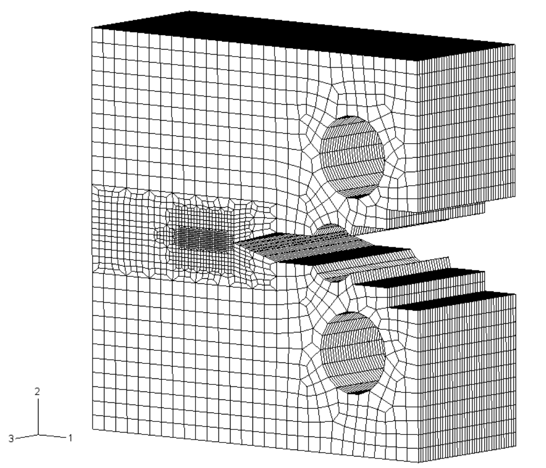

Figure 5: Computational model mesh
Slika 5: Model izračuna

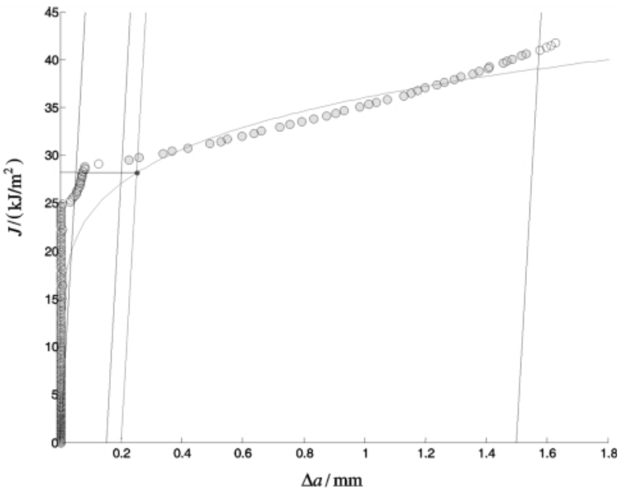

Figure 6: Numerical determination of *J-R* curve
Slika 6: Numerična določitev krivulje J-R

The numerical simulation of the crack growth was performed for CT specimen having dimensions as given in **Figure 4**. The computational model consisting of isoparametric 8-node elements of solid type is shown in **Figure 5**.

The specimen was loaded step-by-step. For each value of the loading force the relevant values of *J*-integral and the crack length increment were determined. The *J*-integral value can be numerically determined by several ways. In some cases the problem is solved as a plane strain problem and the *J*-integral is determined using the relation introduced by Rice [12] (contour integral). Another possibility is using the domain integral introduced by Parks [10] and modified by De Lorenzi [4]. Taking into account [3] and our own experience [9] the *J*-integral value was obtained by the simulation of the experiment given by ASTM standard (see [2], part. 2). In the same way the crack length increment Δa was determined.

The *J-R* curve obtained by the above procedure is presented in **Figure 6**. According to ASTM standard [2] the value of J_{IC} was found to be J_{IC} = 28155 J/m^2.

To obtain the *J-R* curve between exclusion lines the data from 40 points were used. From equation (6) and for k = 1 mm the relation

$$J = C_1\,(\Delta a)^{C_2} \tag{8}$$

where C_1 = 36012.392 and C_2 = 0.178, was obtained.

5 CONCLUSIONS

The presented work presents an approach for the numerical determination of the *J-R* curve. The method – numerical simulation of the experiment for the *J-R* curve determination according to ASTM E standard – was applied to the aluminium alloy.

As a constitutive relation the Complete Gurson Model was used. It is shown that using this model the real behaviour of ductile fracture can be described. The advantage of this method is that for the determination of the *J-R* curve only two types of simple experiments are needed. One for obtaining the stress-strain diagram, which characterises the material behaviour in both the elastic and plastic regions, and, the latter for obtaining the relation loading force *P* versus diameter reduction Δa for the bar of circular cross-section subjected to tensile load. From this relation the necessary parameters for CGM are obtained. It is evident that these experiments can be performed using simple equipment.

ACKNOWLEDGEMENT

The research was carried out within the framework of the grant GA CR 101/03/0731 and the research project MSM 4977751303.

6 REFERENCES

[1] Asta E. P., Chomik E.: Elastic-plastic fracture assessment using a *J-R* curve by direct method. Nuclear Engineering and Design 160, Elsevier, 1996, 129–135

[2] ASTM E 1820-01 Standard Test Method Measurement of Fracture Toughness

[3] Brocks W., Scheider I.: Numerical Aspects of Path-Dependence of *J*-integral in Incremental Plasticity, GKSS-Forschungszentrum Geesthacht, Technical Note GKSS/WMS/01/08, 2001, 33

[4] De Lorenzi H.G.: On the energy release rate and J-integral for 3D crack configurations, J. Fracture 19 (**1982**), 183-193

[5] Dubey J. S., Wadekar S. L., Singh R. N., Sinha T. K., Chakravartty J.K.: Assesment of hydrogen embrittlement of Zircaloy – 2 pressure tubes using unloading compliance and load normalization techniques for determining *J-R* curves, Journal of Nuclear Materials 264 (**1999**), 20–28

[6] Eberle A., Klingbeil D., Schicker J.: The calculation of dynamic -curves from the finite element analysis of a Charpy test using a rate – dependent damage model, Nucleral Engineering and Design 198 (**2000**), 75–87

[7] Koers R. W. J., Krom A. H. M., Bakker A.: Prediction of cleavage fracture in the brittle to ductile transition region of ferritic steel, In Book Constraint Effect in Fracture Theory and Application, 2nd Vol. Eds. M. Kirk, Ad Bakker. STP a STM 1224, 1995

[8] Laš V., Řehounek L., Jaroš P.: Numerical simulation of ductile fracture by Complete Gurson model and comparison with experiment, Proceedings 42nd International Conference Experimental Stress Analysis 2004, Kašperské Hory, Czech Republik, 2004, 137–144

[9] Očenášek J., Laš V.: Contribution to numerical simulation of experiment for estimation of *J-R* curve. Proceedings 19th Conference with International Participation Computational Mechanics 2003, Nečtiny, Czech Republic, 2003, 335–342

[10] Parks D. M.: The virtual crack extension method for nonlinear material behaviour, Computer Methods in Applied Mechanics and Engineering 12 (**1977**), 353–364

[11] Pitard – Bouet J. – M., Seidenfuss M., Bethmont M., Kussmaul K.: Experimental investigations on the "shallow crack effect", on the 10 Mn Mo Ni 55 steel, and computational analysis in the upper shelf by means of the global and local approaches, Nuclear Engineering and Design 190 (**1999**), 171–190

[12] Rice J. R.: A path independent integral and the approximate analysis of strain concentrations by notches and cracks, J. Appl. Mech. 35 (**1968**), 379–386

[13] Tvergaard V., Needleman, A.: Analysis of the cup-cone fracture in round tensile bar, Acta Metalurgica, 32 (**1984**), 157–169

[14] Zadech Gh. M.: Analysis of stable crack growth of a semi-elliptical surface crack by numerical simulation, Nuclear Engineering and Design 158 (**1995**), 311–317

[15] Zhang Z. L.: A complete Gurson model, Nonlinear fracture & Damage mechanics, Editor M. H. Aliabadi, WIT PRESS, 2001, 223–248

[16] Zhu X. K., Jang S. K.: J-R curves corrected by load – independent constraint parameter in ductile crack growth, Engineering Fracture Mechanics 68 (**2001**), 285–301

List of used symbols

a_0 /m – original crack length
a_i /m – current crack length
A_{pl} /J – area corresponding to plastic work
Δa /m – crack length increment
Δa_{min}, Δa_{limit} /m – crack lengths determining feasible domain of the J values
J /(J/m^2) – J-integral
J_{pl} /(J/m^2) – J-integral plastic component
K /(MPa · m$^{1/2}$) – stress intensity factor
B /m – specimen thickness
W /m – specimen width
B_N /m – specimen thickness
Δd /m – notch diameter reduction
f – void volume fraction
q_1, q_2 – constants introduced by Tvergaard
s – standard deviation
E /Pa – Young's modulus
v /m – (load-line) displacement
P /N – force
v_{pl} /m – plastic part of load-line displacement
σ_e /Pa – macroscopic von Mises equivalent stress
σ_k^k /Pa – trace of the macroscopic Cauchy stress tensor
σ_M /Pa – actual yield stress of the matrix material
σ_y /Pa – yield stress
ν – Poisson’s ratio