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A 3-D MATHEMATICAL MODEL OF SOLIDIFICATION OF FEEDING DISTANCES OF CAST-STEEL BARS

3-D MATEMATIČNI MODEL STRJEVANJA IN NAPAJALNE RAZDALJE ZA ULITE JEKLENE PALICE

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A three-dimensional mathematical model of the solidification of cast-steel bars in silica-sand moulds was developed. Fourier's partial differential equation of heat conduction is solved by means of the modified Brian's implicit finite-differences method. The method enables an easier treatment of the boundary conditions and reduces the time of operationalisation of the mathematical model. The thermophysical properties of the materials in the system casting-mould depend on the temperature, which gives nonlinearity to the model. A computer program was written in ASCII FORTRAN and solved on a SPERRY 1106 computer. On the basis of a numerical simulation of the solidification in silica-sand moulds a new formula for the feeding distance as a function of bar thickness is obtained. The results are in relatively good agreement with experimental values from the literature.

Key words: 3-D mathematical model, solidification, feeding distance, cast steel bars

Razvit je bil tridimenzijski matematični model za strjevanje jeklenih palic v formi iz silikatnega peska. Fourierova parcialna diferencialna enačba za toplotno prevodnost je bila rešena z modificirano Brianovo implicitno metodo končnih elementov. Metoda olajša obravnavo robnih pogojev in skrajša čas operacionalizacije matematičnega modela. Termofizikalne lastnosti materialov sistema litja so odvisne od temperature, zato je model nelinearen. Računalniški program je bil napisan v ASCII Fortran in izračunan na računalniku SPERRY 1106. Na osnovi numerične simulacije strjevanja je bila v formi iz silikatnega peska razvita nova odvisnost med razdaljo napajanja in debelino jeklene palice. Rezultati se zadovoljivo ujemajo z empiričnimi podatki iz literature.

Ključne besede: 3-D matematični model, strjevanje, napajalna razdalja, lite jeklene palice

1 INTRODUCTION

The mathematical modeling of the solidification of cast-steel bars in sand moulds can be used to determine the casting feeding distance, which is the sum of the feeding distance of the riser and the feeder. The knowledge of the feeding distance is necessary for the proper positioning of the casting risers and, at present, it is mainly determined empiricaly[1]. Empirical data for the feeding distance of cast-steel bars in literature[2] show a considerable scatter and the feeding distance dependent on the section thickness. The empirical procedure consists of casting of bars of different length and thickness and to determine the length of the shrinkage. This procedure is very time consuming and the results show a considerable scatter because of experimental variables. For this reason, the computed feeding distance for cast-steel bars was determined with numerical simulation of the solidification and in this article a new formula for the feeding distance of low-carbon cast-steel bars as a function of section thickness is proposed.

2 MATHEMATICAL MODEL

The basis of the mathematical model for the solidification of cast-steel bars is the 3-D Fourier's partial differential equation of heat conduction [3].

$$\frac{\partial T}{\partial t} = a(T)\nabla^2 T \tag{1}$$

where $a(T)= k(T)/\rho(T)c_p(T)$ is the thermal diffusivity, which is temperature dependent.

Initial conditions: At time t = 0, the temperature of the sand is T_s, and the temperature of the metal is the pouring temperature T_p. The initial temperature distribution at the sand-metal interface is obtained as analytical solution of the partial differential equation of heat conduction in the case of the thermal contact of two semi-infinite media [4].

$$T_i = T_i + \frac{T_p - T_s}{1 + \frac{k_s}{k_m}\sqrt{\frac{a_m}{a_s}}} \tag{2}$$

Boundary conditions: On the sand-metal interface the boundary condition of the fourth kind is satisfied [4]:

$$k_m \frac{\partial T_m}{\partial_n} = k_s \frac{\partial T_s}{\partial_n} \tag{3}$$

Thermal properties: The thermal properties of the metal and the mould are temperature dependent and have been taken from the literature (Appendix A) [5].

Latent heat of fusion: The latent heat of fusion is evolved in the temperature interval between the liquidus (T_1) and the solidus (T_s), and it is incorporated in the equation for the specific heat of the metal (method of modified specific heat) [(4)].

$$\Delta H_f = \int_{T_s}^{T_1} (c_p^* - c_p) \mathrm{d}T \qquad (4)$$

The liquidus and the solidus temperatures of low-carbon cast steel with 0.25 % C are considered according to empirical data[(5)]: liquidus point at 1499 °C and solidus point at 1449 °C. The latent heat of fusion is of 271 kJ/kg.

3 BRIAN'S IMPLICIT NUMERICAL METHOD

The three-dimensional partial differential equation of heat conduction, with initial and boundary conditions, can be solved using Brian's implicit finite-difference method [6] which is the most efficient method:

$$\delta_x^2 T_{i,j,k}^* + \delta_y^2 T_{i,j,k}^n + \delta_z^2 T_{i,j,k}^n = \frac{1}{a_{i,j,k,n}} \frac{T_{i,j,k}^* - T_{i,j,k}^n}{\Delta t / 2} \qquad (5)$$

$$\delta_x^2 T_{i,j,k}^* + \delta_y^2 T_{i,j,k}^{**} + \delta_z^2 T_{i,j,k}^n = \frac{1}{a_{i,j,k,n}} \frac{T_{i,j,k}^{**} - T_{i,j,k}^n}{\Delta t / 2} \qquad (6)$$

$$\delta_x^2 T_{i,j,k}^* + \delta_y^2 T_{i,j,k}^{**} + \delta_z^2 T_{i,j,k}^{***} = \frac{1}{a_{i,j,k,n}} \frac{T_{i,j,k}^{***} - T_{i,j,k}^n}{\Delta t / 2} \qquad (7)$$

$$\delta_x^2 T_{i,j,k}^* + \delta_y^2 T_{i,j,k}^{**} + \delta_z^2 T_{i,j,k}^{***} = \frac{1}{a_{i,j,k,n}} \frac{T_{i,j,k}^{n+1} - T_{i,j,k}^n}{\Delta t / 2} \qquad (8)$$

In practice, Brian recommended the following simplier form:

$$\delta_x^2 T_{i,j,k}^* + \delta_y^2 T_{i,j,k}^n + \delta_z^2 T_{i,j,k}^n = \frac{1}{a_{i,j,k,n}} \frac{T_{i,j,k}^* - T_{i,j,k}^n}{\Delta t / 2} \qquad (9)$$

$$\delta_y^2 T_{i,j,k}^{**} + \delta_y^2 T_{i,j,k}^n = \frac{1}{a_{i,j,k,n}} \frac{T_{i,j,k}^{**} - T_{i,j,k}^*}{\Delta t / 2} \qquad (10)$$

$$\delta_z^2 T_{i,j,k}^{n+1} + \delta_z^2 T_{i,j,k}^n = \frac{1}{a_{i,j,k,n}} \frac{T_{i,j,k}^{n+1} - T_{i,j,k}^n - 2T_{i,j,k}^{**}}{\Delta t / 2} \qquad (11)$$

It has been proved that the original Brian's method can have also the following form [7]:

$$\delta_x^2 T_{i,j,k}^* + \delta_y^2 T_{i,j,k}^n + \delta_z^2 T_{i,j,k}^n = \frac{1}{a_{i,j,k,n}} \frac{T_{i,j,k}^* - T_{i,j,k}^n}{\Delta t / 2} \qquad (12)$$

$$\delta_x^2 T_{i,j,k}^* + \delta_y^2 T_{i,j,k}^{**} + \delta_z^2 T_{i,j,k}^n = \frac{1}{a_{i,j,k,n}} \frac{T_{i,j,k}^{**} - T_{i,j,k}^n}{\Delta t / 2} \qquad (13)$$

$$\delta_x^2 T_{i,j,k}^* + \delta_y^2 T_{i,j,k}^{**} + \delta_z^2 T_{i,j,k}^{n+1} = \frac{1}{a_{i,j,k,n}} \frac{T_{i,j,k}^{n+1} - T_{i,j,k}^{**}}{\Delta t / 2} \qquad (14)$$

The method is unconditionally stable and converges with a discretization error 0 $[(\Delta x)^2 + (\Delta t)^2]$. It enables an easier treatment of the boundary conditions and reduces the time of operationalisation of the mathematical model. The use of this numerical method results in a system of simultaneous linear algebric equations, which has a tridiagonal form, and it is solved by the Gauss-Jordan method of elimination [8]. Based on the algorithm obtained, a computer program was written in ASCII FORTRAN and solved on a SPERRY 1106 computer.

4 RESULTS AND DISCUSSION

The solidification simulation was performed for cast bars of steel with the following composition: 0.25 % C, 0.40 % Si, 0.60 % Mn and 0.10 % Al. The bars were poured in silica-sand moulds with the composition 90-93 % silica sand, 7–10 % bentonite and 3–5 % water.

The simulation was carried out by space step $\Delta x = \Delta y = \Delta z = 1$ cm and $\Delta x = \Delta y = \Delta z = 2$ cm, and time step $\Delta t = 5$ s and $\Delta t = 10$ s till $t_{max} = 2400$ s. It was concluded that space and time steps do not affect the results of the simulation. The casting temperature was of 1580 °C, the initial temperature of the sand mould was of 25 °C and the initial temperature distribution on the casting-mould interface was of 1540 °C. The diameter and the height of the riser were calculated using a standard procedure [1]. With numerical simulation of the solidification the total feeding distance (TFD) is obtained. This represents the sum of the distance from the end of the bar to the center-line shrinkage (FDE) and from the centerline shrinkage to the riser (FDR). It is schematically represented in **Figure 1**.

A numerical simulation of the solidification was performed for bars of thickness 3 cm to 6 cm, with a step of 1 cm. The total feeding distance was calculated up the cooling time of 95 % of the total solidification time. For low-carbon cast-steel bars the total feeding distance can be represented by the relation obtained using a Hewlett-Packard 48G computer:

$$TFD = 4.1318\ t_t^{0.8375} \qquad (15)$$

where t_t/cm is the thickness of the bar.

The correlation coefficient is 0.98516. The relation for the feeding distance of cast-steel bars poured in silica-sand moulds is shown graphically in **Figure 2**.

The circles in **Figure 2** represent the simulation data, while, the crosses represent experimental data from the

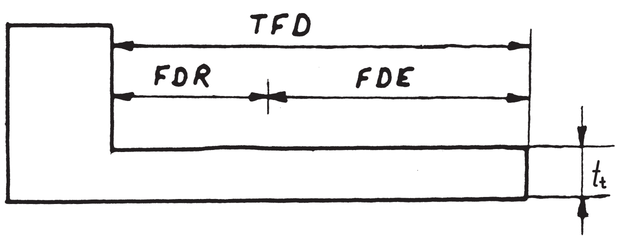

Figure 1: Schematic representation of the feeding distance for the bar with the riser(t_t is the thickness of the bar)

Slika 1: Shema napajalne razdalje za palico z napajalnikom (t_t je debelina palice)

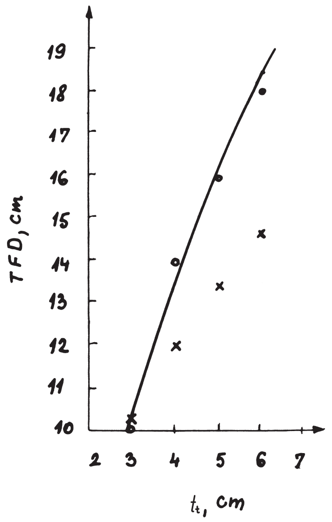

Figure 2: Dependence of total feeding distance (*TFD*) on the bar thickness (t_t)
Slika 2: Odvisnost med skupno dolžino napajanja (*TFD*) in debelino palice (t_t)

literature[2]. It can be concluded that there is relatively good agreement between the simulation and the experimental data, especially for thinner bars.

5 CONCLUSION

A mathematical model of the solidification of low-carbon cast-steel bars poured in silica-sand moulds with known thermophysical properties was formulated and developed in this article. The operationalisation of the model was achieved by means of a modification of the Brian's implicit finite-differences method. On the basis of the solidification simulation a new relation for the feeding distance of steel bars cast in silica-sand moulds, as function of the section thickness, was obtained. The results are in relatively good agreement with experimental data in literature.

Abbreviations used

a – thermal diffusivity
c_p – specific heat capacity at constant pressure
ΔH_f – latent heat of fusion
k – thermal conductivity
n – vertical direction
t – time
T – temperature
x, y, z – space coordinates
t_t – thickness

Appendix A

Thermophysical properties

a) Low-carbon cast steel

Thermal conductivity k/(W/mK):

$T \geq 1499$ °C	$k = 25.96$
1449 °C $> T \geq$ 1449 °C	$k = 207.54 - 0.12114\ T$
1449 °C $> T \geq$ 893 °C	$k = 26.6 + 0.00374\ T$
893 °C $> T$	$k = 50.31 - 0.0225\ T$

Specific heat c_p/(J/kgK):

T ≥ 1499 °C	$c_p = 879.2$
1449 °C > T ≥ 1474 °C	$c_p = 652273.5 - 434.585\ T$
1474 °C > T ≥ 1449 °C	$c_p = 436.258\ T - 631251.9$
1449 °C > T ≥ 982 °C	$c_p = 421.36 + 0.28712\ T$
982 °C > T ≥ 704 °C	$c_p = 1502.8 - 0.81391\ T$
704 °C > T ≥ 427 °C	$c_p = 143.76 + 1.11535\ T$
427 °C > T	$c_p = 458.86 + 0.37681\ T$

b) Mould

Thermal conductivity k/(W/mK):

$$k = 1.35213 \cdot 10^{-7}\ T^2 - 1{,}20052 \cdot 10^{-5}\ T + 0.28254$$

Thermal diffusivity a/(m²/s):

$$a = 2.93581 \cdot 10^{-13}\ T^2 - 2.702541 \cdot 10^{-10}\ T + 2.871021 \cdot 10^{-7}$$

6 LITERATURE

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